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Intrinsic operators for the electromagnetic nuclear current

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Abstract

The intrinsic electromagnetic nuclear meson exchange charge and current operators arising from a separation of the center-of-mass motion are derived for a one-boson-exchange model for the nuclear interaction with scalar, pseudoscalar and vector meson exchange including leading order relativistic terms. Explicit expressions for the meson exchange operators corresponding to the different meson types are given in detail for a two-nucleon system. These intrinsic operators are to be evaluated between intrinsic wave functions in their center-of-mass frame.

I. INTRODUCTION

In a recent paper [1], henceforth called "I", we formally have separated for an arbitrary relativistic transition operator the center-of-mass (c.m.) motion from the intrinsic one. This has been achieved by exploiting the general properties of the Poincaré generators in conjunction with a $1/m$-expansion. As a result, the frame dependence of an arbitrary transition operator has been derived explicitly up to the lowest-order relativistic contributions without reference to any specific dynamic model, leaving undetermined only the genuine intrinsic

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operators. The frame dependent terms have a clear physical meaning describing effects of a Lorentz transformation for a scalar, vector and a general Lorentz tensor of higher rank as well as modifications due to Lorentz contraction and Wigner rotation.

On the other hand, for the determination of the remaining intrinsic operators one needs a specific dynamic model. There exist several techniques for the derivation of the electromagnetic (e.m.) currents using a $1/m$-expansion [2]. In the leading relativistic order, they all lead to unitarily equivalent descriptions, i.e., the various Hamiltonians and e.m. currents are connected by unitary transformations. The e.m. charge and current densities, which we will denote below by $J_0(\vec{k})_{FW}$ and $\vec{J}(\vec{k})_{FW}$ (from Foldy-Wouthuysen), are obtained in a general reference frame in terms of individual nucleon coordinates. In this paper we will start from the e.m. currents obtained in the framework of the extended S-matrix method [3]. They are listed in Appendix A for completeness. The derivation starts from the relativistic Feynman amplitudes from which the iterated Born contribution of the one-nucleon current is subtracted in order to avoid double-counting. The $1/m$-expansion is then employed at the last stage making the technique relatively transparent and easy to use.

In order to write down the transition amplitudes in terms of matrix elements between states with separated c.m. motion, one has to include the effect of the Lorentz boost on the rest frame wave functions, which depend on the individual particle variables [4]. This is done with the help of a unitary transformation which introduces additional so-called boost currents to be added to the FW ones. Then, the c.m. motion can be separated also for the nuclear current operators [1,5], and the transition amplitudes are expressed in terms of matrix elements of intrinsic operators between intrinsic wave functions and simple kinematical factors expressing the c.m. motion effects. The intrinsic currents have a simpler structure than the FW ones and their explicit construction for the one-boson-exchange (OBE) model is the main subject of this paper.

In the next section we first collect the neccessary general expressions as obtained in I. In particular, we give the relations of the intrinsic currents to the FW ones. The boost contributions are written in a convenient form in momentum representation. For simplicity,

we give all explicit expressions for the currents of a two-nucleon system, but the extension to a larger number of nucleons is straightforward. Then, we present in Sect. 3 the intrinsic currents for the one-nucleon and the interaction-dependent meson exchange two-nucleon currents (MEC), corresponding to the exchange of scalar, vector and pseudoscalar mesons, both for isospin 0 and 1. Finally, we summarize our results and give an outlook in Sect. 4.

II. GENERAL CONSIDERATIONS

We will start from the general expressions for a Lorentz vector of "type I" having the leading order in $1/m$ in the zero component as derived in I and [5]. Separating the c.m. motion of the initial and final states, we could write the full operators in terms of pure intrinsic operators where the c.m. motion effects are described by a known functional dependence on $\vec{K} = \vec{P}_f + \vec{P}_i$. Here, $\vec{P}_{i/f}$ denote the total momentum of the initial and final hadronic system, respectively. The intrinsic operators, introduced in I, depend only on the momentum transfer $\vec{k} = \vec{P}_f - \vec{P}_i$ and are denoted by $\rho(\vec{k}\,)$ and $\vec{\jmath}(\vec{k}\,)$ with their nonrelativistic ($\rho^{(0)}(\vec{k}\,)$, $\vec{\jmath}^{\,(1)}(\vec{k}\,)$) and leading order relativistic parts ($\rho^{(2)}(\vec{k}\,)$, $\vec{\jmath}^{\,(3)}(\vec{k}\,)$). Note that the upper index in brackets refers to the order in $1/m$. According to Eqs. (91)-(92) of I, the full operators, which have to be evaluated between intrinsic wave functions only, are given as

$$J_0^{(0)}(\vec{k},\vec{K}) = \rho^{(0)}(\vec{k}\,)\,, \tag{1}$$

$$\vec{J}^{\,(1)}(\vec{k},\vec{K}) = \vec{\jmath}^{\,(1)}(\vec{k}\,) + \frac{\vec{K}}{2M}\,\rho^{(0)}(\vec{k}\,)\,, \tag{2}$$

$$J_0^{(2)}(\vec{k},\vec{K}) = \rho^{(2)}(\vec{k}\,) + \Big(\hat{L} + \hat{W} + \frac{\vec{K}^{\,2}}{8M^2}\Big)\rho^{(0)}(\vec{k}\,) + \frac{1}{2M}\vec{K}\cdot\vec{\jmath}^{\,(1)}(\vec{k}\,)\,, \tag{3}$$

$$\begin{aligned}\vec{J}^{\,(3)}(\vec{k},\vec{K}) = {}& \vec{\jmath}^{\,(3)}(\vec{k}\,) + (\hat{L}+\hat{W})\vec{J}^{\,(1)}(\vec{k},\vec{K}) + \frac{\vec{K}}{8M^2}\vec{K}\cdot\vec{\jmath}^{\,(1)}(\vec{k}\,)\\ &+\frac{\vec{K}}{2M}\Big[\rho^{(2)}(\vec{k}\,) - \frac{1}{2M}\Big(\epsilon_f^{(1)} + \epsilon_i^{(1)} + \frac{\vec{k}^{\,2}}{4M}\Big)\rho^{(0)}(\vec{k}\,)\Big]\,,\end{aligned} \tag{4}$$

where the operators $\hat{L}$ and $\hat{W}$ are defined by

$$\hat{L}f(\vec{k}\,) = -\frac{2\omega_{fi}^{(1)} + \omega_R^{(1)}}{4M}(\vec{K}\cdot\vec{\nabla}^k)f(\vec{k}\,)\,, \tag{5}$$

$$\hat{W}f(\vec{k}\,) = -\frac{i}{8M^2}\Big\{\vec{S}\cdot(\vec{k}\times\vec{K})\,,f(\vec{k}\,)\Big\}\,. \tag{6}$$

Here M denotes the total mass of the system and f stands for ρ or $\vec{\jmath}$. The gradient $\vec{\nabla}^k$ *does not* act on the nucleon form factors [1,5]. $\vec{S}$ is the total spin operator of the composite system. The nonrelativistic intrinsic excitation energy is denoted by

$$\omega_{fi}^{(1)} = \epsilon_f^{(1)} - \epsilon_i^{(1)} \,, \tag{7}$$

with $\epsilon_{i/f}^{(1)}$ as the nonrelativistic energies for the initial and final states, respectively. Finally, $\omega_R^{(1)}$ is the nonrelativistic recoil energy

$$\omega_R^{(1)} = \frac{\vec{P}_f^{\,2}}{2M} - \frac{\vec{P}_i^{\,2}}{2M} = \frac{(\vec{k}\cdot\vec{K})}{2M} \,. \tag{8}$$

The $\hat{L}$-term describes the Lorentz contraction of the system, while $\hat{W}$ reflects the Wigner rotation of the total spin associated with the transformation from the Breit frame ($\vec{P}_f = -\vec{P}_i = \vec{k}/2$) to a general one [5,6]. Note, that in k-congruent frames, i.e., those for which $\vec{P}_i$ and thus $\vec{K}$ are parallel to $\vec{k}$, the $\hat{W}$-term vanishes. The $\hat{L}$-term can be absorbed into the nonrelativistic operator by replacing $\vec{k}$ by an effective momentum transfer $\vec{k}_{eff}$ having the same direction as $\vec{k}$ and with $\vec{k}_{eff}^{\,2} = \vec{k}^{\,2} - (k_0^{(1)})^2 + (\omega_{fi}^{(1)})^2)$ [1]. The intrinsic operators, introduced in I, are obtained from the expressions (1)–(4) if taken in the Breit frame, i.e.,

$$j_\lambda(\vec{k}\,) = J_\lambda(\vec{k},\vec{0}) \,. \tag{9}$$

Except for the $\hat{L}$ and $\hat{W}$ terms, all other contributions in (1)-(4) can be obtained by the Lorentz transformation of the charge and current operators from the Breit frame to a general one. The parameters $\vec{\beta}$ and γ of such a transformation are given by

$$\vec{\beta} = \frac{\vec{K}}{E_f + E_i} \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} \,. \tag{10}$$

For an arbitrary k-congruent frame, one can generalize the expressions in (1)-(4) to

$$J_0(\vec{k},\vec{K}) = \gamma\Big(\rho(\vec{k}_{eff}) + \vec{\beta}\cdot\vec{\jmath}(\vec{k}_{eff})\Big) \,, \tag{11}$$

$$\vec{J}(\vec{k},\vec{K}) = \Big(\vec{\jmath}(\vec{k}_{eff}) + \frac{\gamma-1}{\beta^2}(\vec{\beta}\cdot\vec{\jmath}(\vec{k}_{eff}))\vec{\beta} + \gamma\vec{\beta}\,\rho(\vec{k}_{eff})\Big) \,. \tag{12}$$

Then all kinematical effects related to the Lorentz vector structure of the current are taken into account exactly. Only in the intrinsic charge and current densities remain approximations with respect to the $1/m$-expansion in the derivation and with the introduction of the effective momentum transfer $\vec{k}_{eff}$.

As described in detail in I, the current operator $J_\lambda(\vec{k},\vec{K})$ acting in the space of intrinsic wave functions is defined by the matrix element of the e.m. operators between plane waves describing the c.m. motion of the system

$$\langle\vec{P}_f|J_\lambda(\vec{k}\,)|\vec{P}_i\rangle = \Big(\frac{M_f M_i}{E_f E_i}\Big)^{1/2} J_\lambda(\vec{k},\vec{K})\,\delta(\vec{P}_f-\vec{P}_i-\vec{k}\,)\,, \tag{13}$$

with nonrelativistic normalization of the plane waves

$$\langle\vec{P}_f|\vec{P}_i\rangle = \delta(\vec{P}_f-\vec{P}_i)\,. \tag{14}$$

Therefore, the factor in front of $J_\lambda(\vec{k},\vec{K})$ in (13) ensures that $J_0(\vec{k},\vec{K})$ and $\vec{J}(\vec{k},\vec{K})$ represent *covariantly* normalized operators in the space of intrinsic wave functions. We have chosen the normalization convention (14) since it is usually adopted implicitly in the derivations of the FW operators on which the construction of the intrinsic operators is based. For this reason we introduce in addition *noncovariantly* normalized operators of the full current by

$$\tilde{J}_\lambda(\vec{k},\vec{K}) = \Big(\frac{M_f M_i}{E_f E_i}\Big)^{1/2} J_\lambda(\vec{k},\vec{K})\,. \tag{15}$$

Up to the order considered here, the full charge and current operators on the l.h.s. of (13) are given by

$$J_\lambda(\vec{k}\,) = J_\lambda(\vec{k}\,)_{FW} + i\left[\chi\,, J_\lambda(\vec{k}\,)_{FW}\right], \tag{16}$$

where χ describes the wave function boost [4]. The commutator term is usually called the boost contribution to the charge and current.

The derivation of the intrinsic currents as well as their final form simplify considerably, if one splits their relativistic parts in the following way, employed implicitly by Friar, Gibson and Payne [5],

$$\rho^{(2)}(\vec{k}\,) = \rho_F^{(2)}(\vec{k}\,) - \frac{\vec{k}^{\,2}}{8M^2}(\vec{k}\cdot\vec{\nabla}^k)\rho^{(0)}(\vec{k}\,) + \frac{\vec{k}^{\,2}}{8M^2}\rho^{(0)}(\vec{k}\,)\,, \tag{17}$$

$$\vec{\jmath}^{\,(3)}(\vec{k}\,) = \vec{\jmath}_F^{\,(3)}(\vec{k}\,) - \frac{\vec{k}^{\,2}}{8M^2}(\vec{k}\cdot\vec{\nabla}^k)\vec{\jmath}^{\,(1)}(\vec{k}\,)\,. \tag{18}$$

The gradient terms can then be absorbed in the operator $\hat{L}$ leading in turn to a new effective momentum transfer $\vec{k}_F$ given by

$$\vec{k}_F^{\,2} = \vec{k}^{\,2} - (k_0^{(1)})^2 + (\omega_{fi}^{(1)})^2 - \frac{\vec{k}^{\,4}}{4M^2}\,, \tag{19}$$

where the direction of $\vec{k}_F$ is again parallel to $\vec{k}$. Note, that $\vec{k}_F^{\,2}$ is still effectively a Lorentz scalar, up to the order considered. The introduction of this new effective momentum transfer $\vec{k}_F$ leads to a rearrangement of the intrinsic operators in the following way

$$\rho^{(0)}(\vec{k}\,) \rightarrow \rho^{(0)}(\vec{k}_F)\,, \tag{20}$$

$$\vec{\jmath}^{\,(1)}(\vec{k}\,) \rightarrow \vec{\jmath}^{\,(1)}(\vec{k}_F)\,, \tag{21}$$

$$\rho^{(2)}(\vec{k}\,) \;=\; \rho_F^{(2)}(\vec{k}\,) + \frac{\vec{k}^{\,2}}{8M^2}\rho^{(0)}(\vec{k}\,)\,, \tag{22}$$

$$\vec{\jmath}^{\,(3)}(\vec{k}\,) \;=\; \vec{\jmath}_F^{\,(3)}(\vec{k}\,)\,. \tag{23}$$

Note, that now part of the relativistic effects are contained in $\rho^{(0)}$ and $\vec{\jmath}^{\,(1)}$.

Therefore, it is sufficient to determine the operators ρ_F and $\vec{\jmath}_F$. In terms of noncovariantly normalized operators, they are in general given by

$$\rho_F^{(0)}(\vec{k}\,) = \tilde{J}_0^{(0)}(\vec{k},0)\,, \tag{24}$$

$$\vec{\jmath}_F^{\,(1)}(\vec{k}\,) = \tilde{\vec{J}}^{(1)}(\vec{k},0)\,, \tag{25}$$

$$\rho_F^{(2)}(\vec{k}\,) = \tilde{J}_0^{(2)}(\vec{k},0) + \rho_{sep}^{(2)}(\vec{k}\,)\,, \tag{26}$$

$$\vec{\jmath}_F^{\,(3)}(\vec{k}\,) = \tilde{\vec{J}}^{(3)}(\vec{k},0) + \vec{\jmath}_{sep}^{\,(3)}(\vec{k}\,)\,, \tag{27}$$

where we have introduced the separation charge and current operators

$$\rho_{sep}^{(2)}(\vec{k}\,) = \frac{\vec{k}^{\,2}}{8M^2}(\vec{k}\cdot\vec{\nabla}^k)\,\rho_F^{(0)}(\vec{k}\,)\,, \tag{28}$$

$$\vec{\jmath}_{sep}^{\,(3)}(\vec{k}\,) = \frac{\vec{k}^{\,2}}{8M^2}\Big(1 + (\vec{k}\cdot\vec{\nabla}^k)\Big)\,\vec{\jmath}_F^{\,(1)}(\vec{k}\,)\,, \tag{29}$$

and $\rho_F^{(0)}(\vec{k}\,) = \rho^{(0)}(\vec{k}\,)$ and $\vec{\jmath}_F^{\,(1)}(\vec{k}\,) = \vec{\jmath}^{\,(1)}(\vec{k}\,)$ in order to unify our notation.

The e.m. current should satisfy the continuity equation, which means in our notation

$$\vec{k} \cdot \vec{J}(\vec{k}, \vec{K}) = k_0 J_0(\vec{k}, \vec{K}) \,. \tag{30}$$

According to I, it implies for the intrinsic operators the following relations

$$\vec{k} \cdot \vec{\jmath}_F^{\,(1)}(\vec{k}\,) = \Big[h^{(1)},\, \rho_F^{(0)}(\vec{k}\,)\Big] \,, \tag{31}$$

$$\vec{k} \cdot \vec{\jmath}_F^{\,(3)}(\vec{k}\,) = \Big[h^{(1)},\, \rho_F^{(2)}(\vec{k}\,)\Big] + \Big[h^{(3)} - \frac{\vec{k}^{\,2}}{8M^2} h^{(1)},\, \rho_F^{(0)}(\vec{k}\,)\Big] \,. \tag{32}$$

where h denotes the intrinsic Hamiltonian. It is useful to separate the contributions of the one-body and the meson exchange currents, denoted by $a(1;k)$ and $a(2;k)$, respectively. Splitting h into kinetic and potential energy $h = t + v$, one would expect for the one-body charge and current operators of the two-nucleon system ($M = 2m$ denoting by m the nucleon mass) to satisfy the relations

$$\vec{k} \cdot \vec{\jmath}_F^{\,(1)}(1; \vec{k}) = \Big[t^{(1)},\, \rho_F^{(0)}(1; \vec{k})\Big] \,, \tag{33}$$

$$\vec{k} \cdot \vec{\jmath}_F^{\,(3)}(1; \vec{k}) = \Big[t^{(1)},\, \rho_F^{(2)}(1; \vec{k})\Big] + \Big[t^{(3)} - \frac{\vec{k}^{\,2}}{32m^2} t^{(1)},\, \rho_F^{(0)}(1; \vec{k})\Big] \,. \tag{34}$$

Consequently, the two-body MEC operators would have to fulfil

$$\vec{k} \cdot \vec{\jmath}_F^{\,(1)}(2; \vec{k}) = \Big[v^{(1)},\, \rho_F^{(0)}(1; \vec{k})\Big] \,, \tag{35}$$

$$\vec{k} \cdot \vec{\jmath}_F^{\,(3)}(2; \vec{k}) = \Big[h^{(1)},\, \rho_F^{(2)}(2; \vec{k})\Big] + \Big[v^{(1)},\, \rho_F^{(2)}(1; \vec{k})\Big] + \Big[v^{(3)} - \frac{\vec{k}^{\,2}}{32m^2} v^{(1)},\, \rho_F^{(0)}(1; \vec{k})\Big] \,, \tag{36}$$

where we already have used the fact that $\rho_F^{(0)}(2; \vec{k})$ vanishes. However, the relations (34) and (36) will be slightly modified in Sect. III.A (see (67) and (68)), because first of all we will incorporate into the one-body current $\vec{\jmath}_F^{\,(3)}(1; \vec{k})$ a two-body part effectively, and secondly, the first commutator on the r.h.s. of (36) will contain only $t^{(1)}$ because the commutator with $v^{(1)}$ will be of higher order in the meson nucleon coupling constants not considered here.

In order to get the intrinsic charge and current densities from (24) through (27) one needs the FW and boost contributions expressed in the Breit frame. The FW currents are

listed in the appendices and their evaluation in the Breit frame is straightforward. The wave function boost χ contains in general a kinetic χ_0 and an interaction dependent part χ_V. In the OBE model, the interaction boost is non-zero only for a pseudoscalar exchange interaction. It is dealt with explicitly in Appendix C. The leading order kinetic contribution ($\sim 1/m^2$) reads in terms of c.m. and relative variables

$$\chi_0 = -\frac{1}{2}\sum_{a=1}^{A}\frac{(\vec{\rho}_a\cdot\vec{P})(\vec{\pi}_a\cdot\vec{P})+h.c.}{2M^2} - \frac{1}{2}\sum_{a=1}^{A}\frac{(\vec{\rho}_a\cdot\vec{P})(\vec{\pi}_a{}^2)+h.c.}{2Mm_a} + \sum_{a=1}^{A}\frac{\vec{s}_a\times\vec{\pi}_a\cdot\vec{P}}{2Mm_a}\,, \qquad (37)$$

where $\vec{\rho}_a = \vec{r}_a - \vec{R}$, $\vec{\pi}_a = \vec{p}_a - \frac{m_a}{M}\vec{P}$, $\vec{P}$ is the total momentum operator and $\vec{R}$ the c.m. coordinate given by the usual expression

$$\vec{R} = \sum_{a=1}^{A} m_a\vec{r}_a/M\,. \qquad (38)$$

For two particles with equal mass (m), the second term of (37) vanishes and one gets

$$\chi_0 = -\frac{1}{16m^2}\Big((\vec{r}\cdot\vec{P})(\vec{p}\cdot\vec{P}) + (\vec{p}\cdot\vec{P})(\vec{r}\cdot\vec{P})\Big) + \frac{1}{8m^2}(\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{p}\cdot\vec{P}\,, \qquad (39)$$

where $\vec{r} = \vec{r}_1 - \vec{r}_2$ and $\vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$. Let us now evaluate the boost commutator in the Breit frame using the momentum representation. To this end, we denote an intrinsic operator $a(\vec{k})$ in momentum representation by

$$a(\vec{k},\vec{q},\vec{Q}) = \langle\vec{p}\,'|a(\vec{k})|\vec{p}\rangle\,, \qquad (40)$$

where $\vec{q} = \vec{p}\,' - \vec{p}$ and $\vec{Q} = \vec{p}\,' + \vec{p}$. Noting, that the boost operator χ is diagonal with respect to the c.m. plane waves, i.e.,

$$\langle\vec{P}\,'\,|\,\chi\,|\vec{P}\,\rangle = \chi(\vec{P},\vec{r},\vec{p})\,\delta(\vec{P}\,'-\vec{P})\,, \qquad (41)$$

one finds, that the intrinsic commutator of the kinetic boost with an operator $a(\vec{k})$ reads

$$\langle\,\vec{p}\,'|\,i\Big(\chi_0(\frac{\vec{k}}{2},\vec{r},\vec{p})\,a(\vec{k})\, - a(\vec{k})\,\chi_0(-\frac{\vec{k}}{2},\vec{r},\vec{p})\,\Big)|\vec{p}\rangle = a_{\chi_r}(\vec{k},\vec{q},\vec{Q}) + a_{\chi_\sigma}(\vec{k},\vec{q},\vec{Q})\,, \qquad (42)$$

where

$$a_{\chi_r}(\vec{k},\vec{q},\vec{Q}) = \frac{1}{32m^2}\Big(\vec{k}^{\,2} + (\vec{k}\cdot\vec{q})\,(\vec{k}\cdot\vec{\nabla}^q) + (\vec{k}\cdot\vec{Q})\,(\vec{k}\cdot\vec{\nabla}^Q)\Big)\,a(\vec{k},\vec{q},\vec{Q})\,, \tag{43}$$

$$\begin{aligned} a_{\chi_\sigma}(\vec{k},\vec{q},\vec{Q}) = \frac{i}{32m^2}\Big(&\Big\{a(\vec{k},\vec{q},\vec{Q}),\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\Big\} \\ &-\Big[a(\vec{k},\vec{q},\vec{Q}),\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{q}\times\vec{k})\Big]\Big)\,. \end{aligned} \tag{44}$$

Since the FW currents are usually given in momentum representation, the boost contributions follow directly from (42). The extension to systems with more than two nucleons is straightforward. The plane wave basis in the intrinsic space can be constructed with respect to the set of Jacobi coordinates and, employing their commutator relations, one can derive an analogue of (42). In this case, the contribution of the second term of (37) has to be included.

III. INTRINSIC CURRENTS

In this section, explicit expressions are given for the intrinsic currents ρ_F and $\vec{\jmath}_F$ of the two-nucleon system as they follow from (24)-(27). For the two-body currents, we distinguish different terms $a_F^{(n)}(2;\beta;\vec{k},\vec{q},\vec{Q})_B^{iso}$, labelled by β (= pro, mes, and ret), according to their meson propagator structure. Furthermore, we denote the isospin structure of the MECs by the superscript "*iso*" (= +, −), and by the subscript "*B*", the exchanged meson type. Each of the currents may be decomposed in general according to the different contributions arising from the FW-part, the boost and the so-called separation part. Labelling them by α (= *FW*, χ_r, χ_σ, and *sep*), this reads in momentum space

$$a_F^{(n)}(1;\vec{k},\vec{q},\vec{Q}) = \sum_\alpha a_\alpha^{(n)}(1;\vec{k},\vec{q},\vec{Q})\,, \tag{45}$$

$$a_F^{(n)}(2;\beta;\vec{k},\vec{q},\vec{Q})_B^{iso} = \sum_\alpha a_\alpha^{(n)}(2;\beta;\vec{k},\vec{q},\vec{Q})_B^{iso}\,. \tag{46}$$

Further details of our notation are explained in Appendix A.

For each case (one-nucleon or MEC operator) we will begin with the FW terms in the Breit frame, then consider the boost commutators (42) and finally the additional terms from the r.h.s. of (26)-(27). The latter ones are referred to as "separation" operators and labelled

by "*sep*" as already introduced above. Finally, the total intrinsic currents are listed in a way which makes clear which part of the intrinsic continuity equations (33)-(36) or their modified forms (65)-(68) they saturate.

However, if the corresponding nonrelativistic operator does not exist, as in the case of the interaction dependent charge densities or "+" parts of MECs, the intrinsic operators ρ_F and $\vec{\jmath}_F$ are simply equal to the FW ones taken in the Breit frame. In such cases, we do not write them down repeatedly, but list them together with other intrinsic operators. Also, some nonrelativistic operators do not depend on the momentum $\vec{Q}$ which further simplifies our notation.

A. One-nucleon currents

We will consider explicitly the currents of the nucleon labelled "1", while those of the second one follow by a replacement $(1 \leftrightarrow 2)$. This replacement, of course, affects also the relative variables introduced above, changing the sign of $\vec{r}, \vec{p}, \vec{p}\,', \vec{q},$ and $\vec{Q}$.

For the nonrelativistic operators, one finds immediately from (A.14) and (A.16) in the Breit frame

$$\rho_F^{(0)}(1;\vec{k},\vec{q})_1 = \hat{e}_1\, \delta(\frac{\vec{k}}{2} - \vec{q})\,, \tag{47}$$

$$\begin{aligned} \vec{\jmath}_F^{\,(1)}(1;\vec{k},\vec{q},\vec{Q})_1 &= \frac{1}{2m}\Big(\hat{e}_1\,\vec{Q} + i(\hat{e}_1 + \hat{\kappa}_1)\,\vec{\sigma}_1 \times \vec{k}\Big)\,\delta(\frac{\vec{k}}{2} - \vec{q}) \\ &= \vec{\jmath}_c^{\,(1)}(1;\vec{k},\vec{q},\vec{Q})_1 + \vec{\jmath}_s^{\,(1)}(1;\vec{k},\vec{q})_1\,, \end{aligned} \tag{48}$$

where $\vec{\jmath}_{c,s}^{\,(1)}$ stand for the usual nonrelativistic convection and spin currents.

Now we turn to the relativistic contributions. We first note that, since the currents of the first nucleon contain $\delta(\vec{k}/2 - \vec{q})$, the contributions to the boost commutator (42) simplify to

$$a_{\chi_r}(1;\vec{k},\vec{q},\vec{Q})_1 = \frac{1}{32m^2}\Big(\frac{\vec{k}^{\,2}}{2}(\vec{k}\cdot\vec{\nabla}^q) + (\vec{k}\cdot\vec{Q})\,(\vec{k}\cdot\vec{\nabla}^Q)\Big)\,a(1;\vec{k},\vec{q},\vec{Q})_1\,, \tag{49}$$

$$a_{\chi_\sigma}(1;\vec{k},\vec{q},\vec{Q})_1 = \frac{i}{32m^2}\Big\{a(1;\vec{k},\vec{q},\vec{Q})_1\,,\,(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\Big\}\,. \tag{50}$$

For the FW part of the charge density, one obtains from (A.15) in the Breit frame

$$\rho^{(2)}_{FW}(1;\vec{k},\vec{q},\vec{Q})_1 = -\frac{\hat{e}_1+2\hat{\kappa}_1}{8m^2}\Big(\vec{k}^{\,2}+i\vec{\sigma}_1\cdot(\vec{Q}\times\vec{k})\Big)\delta(\frac{\vec{k}}{2}-\vec{q}\,)\,. \tag{51}$$

The boost and separation contributions to the charge density are

$$\rho^{(2)}_{\chi_r}(1;\vec{k},\vec{q},\vec{Q})_1 = -\frac{\hat{e}_1\,\vec{k}^{\,2}}{32m^2}\,(\vec{k}\cdot\vec{\nabla}^k)\delta(\frac{\vec{k}}{2}-\vec{q}\,)\,, \tag{52}$$

$$\rho^{(2)}_{\chi_\sigma}(1;\vec{k},\vec{q},\vec{Q})_1 = i\frac{\hat{e}_1}{16m^2}\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\delta(\frac{\vec{k}}{2}-\vec{q}\,)\,, \tag{53}$$

$$\rho^{(2)}_{sep}(1;\vec{k},\vec{q},\vec{Q})_1 = \frac{\hat{e}_1\,\vec{k}^{\,2}}{32m^2}\,(\vec{k}\cdot\vec{\nabla}^k)\delta(\frac{\vec{k}}{2}-\vec{q}\,)\,. \tag{54}$$

The expressions (52) and (54) cancel completely. This is, of course, the reason why $\rho_F(\vec{k}\,)$ and $\vec{\jmath}_F(\vec{k}\,)$ are introduced in (17)-(18). The relativistic part of the intrinsic one-nucleon charge density is then

$$\begin{aligned}\rho^{(2)}_{F}(1;\vec{k},\vec{q},\vec{Q})_1 =& -\frac{\hat{e}_1}{16m^2}\Big(2\vec{k}^{\,2}+i(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\Big)\delta(\frac{\vec{k}}{2}-\vec{q}\,)\\ &-\frac{\hat{\kappa}_1}{4m^2}\Big(\vec{k}^{\,2}+i\vec{\sigma}_1\cdot(\vec{Q}\times\vec{k})\Big)\delta(\frac{\vec{k}}{2}-\vec{q}\,)\\ =&\ \rho^{(2)}_{F,e}(1;\vec{k},\vec{q},\vec{Q})_1+\rho^{(2)}_{F,\kappa}(1;\vec{k},\vec{q},\vec{Q})_1\,.\end{aligned} \tag{55}$$

In a similar way, one gets for the spatial current

$$\begin{aligned}\vec{\jmath}^{\,(3)}_{FW}(1;\vec{k},\vec{q},\vec{Q})_1 =& -\frac{1}{16m^3}\Big[(\vec{Q}^{\,2}+\vec{k}^{\,2})\Big(\hat{e}_1\,\vec{Q}+i(\hat{e}_1+\hat{\kappa}_1)\,\vec{\sigma}_1\times\vec{k}\Big)\\ &+\Big(\hat{e}_1\,(\vec{k}\cdot\vec{Q})+4\hat{\kappa}_1 m\omega^{(1)}_{fi}\Big)\,(\vec{k}+i\vec{\sigma}_1\times\vec{Q})\\ &+\hat{\kappa}_1\,\vec{k}\times[\vec{Q}\times(\vec{k}+i\vec{\sigma}_1\times\vec{Q})]\Big]\delta(\frac{\vec{k}}{2}-\vec{q}\,)\,,\end{aligned} \tag{56}$$

$$\begin{aligned}\vec{\jmath}^{\,(3)}_{\chi_r}(1;\vec{k},\vec{q},\vec{Q})_1 =& -\frac{\vec{k}^{\,2}}{32m^2}\,(\vec{k}\cdot\vec{\nabla}^k)\vec{\jmath}^{\,(1)}_{F}(1;\vec{k},\vec{q},\vec{Q})_1\\ &+\frac{1}{32m^2}\Big(\vec{k}\,(\vec{k}\cdot\vec{\jmath}^{\,(1)}_{c}(1;\vec{k},\vec{q},\vec{Q})_1)+\vec{k}^{\,2}\,\vec{\jmath}^{\,(1)}_{s}(1;\vec{k},\vec{q}\,)_1\Big)\,,\end{aligned} \tag{57}$$

$$\begin{aligned}\vec{\jmath}^{\,(3)}_{\chi_\sigma}(1;\vec{k},\vec{q},\vec{Q})_1 =& \frac{1}{32m^3}\Big[i\hat{e}_1\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\,\vec{Q}\\ &-(\hat{e}_1+\hat{\kappa}_1)\,[\vec{k}\times(\vec{k}\times\vec{Q})+\vec{\sigma}_2\cdot(\vec{Q}\times\vec{k})(\vec{k}\times\vec{\sigma}_1)]\Big]\delta(\frac{\vec{k}}{2}-\vec{q}\,)\,,\end{aligned} \tag{58}$$

$$\vec{\jmath}^{\,(3)}_{sep}(1;\vec{k},\vec{q},\vec{Q})_1 = \frac{\vec{k}^{\,2}}{32m^2}\Big(1+(\vec{k}\cdot\vec{\nabla}^k)\Big)\vec{\jmath}^{\,(1)}_{F}(1;\vec{k},\vec{q},\vec{Q})_1\,. \tag{59}$$

There is again a cancellation between the separation and boost terms. The final expression for the relativistic part of the one-nucleon intrinsic current operator then is

$$
\begin{aligned}
\vec{\jmath}_F^{(3)}(1;\vec{k},\vec{q},\vec{Q})_1 = &-\frac{1}{16m^3}\Big[\hat{e}_1(\vec{Q}^{\,2}+\frac{\vec{k}^{\,2}}{4})\vec{Q}+i(\hat{e}_1+\hat{\kappa}_1)(\vec{Q}^{\,2}+\frac{\vec{k}^{\,2}}{2})\,\vec{\sigma}_1\times\vec{k} \\
&+(\hat{e}_1\,(\vec{k}\cdot\vec{Q})+4\hat{\kappa}_1 m\omega_{fi}^{(1)})(\vec{k}+i\vec{\sigma}_1\times\vec{Q}) \\
&-\frac{i}{2}\hat{e}_1\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\,\vec{Q}+\frac{1}{2}(\hat{e}_1+\hat{\kappa}_1)\vec{\sigma}_2\cdot(\vec{Q}\times\vec{k})(\vec{k}\times\vec{\sigma}_1) \\
&+\hat{\kappa}_1\,\vec{k}\times[\vec{Q}\times(\frac{1}{2}\vec{k}+i\vec{\sigma}_1\times\vec{Q})]\Big]\delta(\frac{\vec{k}}{2}-\vec{q}\,) \\
&-\frac{1}{32m^2}\vec{k}\,(\vec{k}\cdot\vec{\jmath}_F^{(1)}(1;\vec{k},\vec{q},\vec{Q})_1)\,.
\end{aligned}
\tag{60}
$$

Now we will consider the divergence of these currents in view of the general continuity equations for the intrinsic operators in (31) and (32). Using $\vec{k}=2\vec{q}$, one finds for the divergence of the nonrelativistic intrinsic current

$$
\vec{k}\cdot\vec{\jmath}_F^{(1)}(1;\vec{k},\vec{q},\vec{Q})_1=\frac{(\vec{q}\cdot\vec{Q})}{m}\,\rho_F^{(0)}(1;\vec{k},\vec{q}\,)_1\,.
\tag{61}
$$

and for the relativistic one

$$
\begin{aligned}
\vec{k}\cdot\vec{\jmath}_F^{(3)}(1;\vec{k},\vec{q},\vec{Q})_1 = &-\frac{1}{16m^3}\Big[2\hat{e}_1(\vec{Q}^{\,2}+\frac{\vec{k}^2}{4})(\vec{q}\cdot\vec{Q})+\hat{e}_1(2\vec{k}^{\,2}+i(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k}))(\vec{q}\cdot\vec{Q}) \\
&+4\hat{\kappa}_1 m\omega_{fi}^{(1)}(\vec{k}^{\,2}+i\vec{\sigma}_1\cdot(\vec{Q}\times\vec{k}))\Big]\delta(\frac{\vec{k}}{2}-\vec{q}\,)-\frac{\vec{k}^{\,2}}{32m^2}\,(\vec{k}\cdot\vec{\jmath}_F^{(1)}(1;\vec{k},\vec{q},\vec{Q})_1) \\
= &-\frac{1}{8m^3}(\vec{Q}^{\,2}+\vec{q}^{\,2})(\vec{q}\cdot\vec{Q})\rho_F^{(0)}(1;\vec{k},\vec{q}\,)_1+\frac{1}{m}(\vec{q}\cdot\vec{Q})\rho_{F,e}^{(2)}(1;\vec{k},\vec{q},\vec{Q})_1 \\
&+\omega_{fi}^{(1)}\rho_{F,\kappa}^{(2)}(1;\vec{k},\vec{q},\vec{Q})_1-\frac{\vec{k}^{\,2}}{32m^3}\,\frac{(\vec{q}\cdot\vec{Q})}{m}\,\rho_F^{(0)}(1;\vec{k},\vec{q}\,)_1\,,
\end{aligned}
\tag{62}
$$

where $\rho_{F,e}^{(2)}(1;\vec{k}\,)$ and $\rho_{F,\kappa}^{(2)}(1;\vec{k}\,)$ are defined in (55).

Since for an intrinsic operator in momentum representation $a(\vec{q},\vec{Q})$ the following relations hold

$$
\langle\vec{p}\,'|\Big[t^{(1)},a\Big]|\vec{p}\,\rangle=\frac{(\vec{q}\cdot\vec{Q})}{m}a(\vec{q},\vec{Q})\,,
\tag{63}
$$

$$
\langle\vec{p}\,'|\Big[t^{(3)},a\Big]|\vec{p}\,\rangle=-\frac{(\vec{q}\cdot\vec{Q})}{8m^3}(\vec{Q}^{\,2}+\vec{q}^{\,2})a(\vec{q},\vec{Q})\,,
\tag{64}
$$

one finds that the r.h.s. of (61) is just the commutator of the nonrelativistic intrinsic kinetic energy $\vec{p}^{\,2}/m$ with the charge density

$$
\vec{k}\cdot\vec{\jmath}_F^{(1)}(1;\vec{k},\vec{q},\vec{Q})_1=\langle\vec{p}\,'|\Big[t^{(1)},\rho_F^{(0)}(1;\vec{k}\,)_1\Big]|\vec{p}\,\rangle\,,
\tag{65}
$$

i.e. the relation (33), and for the divergence of the relativistic contribution in (62)

$$\vec{k}\cdot\vec{\jmath}_F^{\,(3)}(1;\vec{k},\vec{q},\vec{Q})_1 = \langle\vec{p}\,'|\Big[t^{(1)},\rho^{(2)}_{F,e}(1;\vec{k})_1\Big]|\vec{p}\,\rangle + \langle\vec{p}\,'|\Big[h^{(1)},\rho^{(2)}_{F,\kappa}(1;\vec{k})_1\Big]|\vec{p}\,\rangle$$
$$+\langle\vec{p}\,'|\Big[t^{(3)} - \frac{\vec{k}^{\,2}}{32m^2}\,t^{(1)},\rho^{(0)}_F(1;\vec{k})_1\Big]|\vec{p}\,\rangle\,, \qquad (66)$$

which almost equals (34) with the sole exception, that in the commutator of $\rho^{(2)}_{F,\kappa}(1;\vec{k})$ the full nonrelativistic intrinsic Hamiltonian $h^{(1)}$ appears instead of the kinetic energy $t^{(1)}$. This modification of (34), we had already alluded to, is a consequence of the fact that we have kept in the $\hat{\kappa}$-part of $\vec{\jmath}_F^{\,(3)}(1;\vec{k})$ in (60) the total intrinsic energy transfer, thus including implicitly a two-body contribution.

That means, that in order to satisfy the full continuity equation for the intrinsic currents, one should find for the intrinsic model-dependent interaction currents the following relations involving commutators with the NN potential v

$$\vec{k}\cdot\vec{\jmath}_F^{\,(1)}(2;\vec{k},\vec{q},\vec{Q}) = \langle\vec{p}\,'\,|\Big[v^{(1)}\,,\,\rho^{(0)}_F(1;\vec{k}\,)\Big]|\vec{p}\,\rangle\,, \qquad (67)$$

$$\vec{k}\cdot\vec{\jmath}_F^{\,(3)}(2;\vec{k},\vec{q},\vec{Q}) = \langle\vec{p}\,'\,|\Big[t^{(1)},\rho^{(2)}_F(2;\vec{k})\Big] + \Big[v^{(1)}\,,\,\rho^{(2)}_{F,e}(1;\vec{k}\,)\Big]|\vec{p}\,\rangle$$
$$+\langle\vec{p}\,'\,|\Big[v^{(3)} - \frac{\vec{k}^{\,2}}{32m^2}v^{(1)},\,\rho^{(0)}_F(1;\vec{k}\,)\Big]|\vec{p}\,\rangle\,, \qquad (68)$$

where the last term is the interaction-dependent part of the recoil commutator in (32). Note, that only $\rho^{(2)}_{F,e}(1;\vec{k}\,)$ appears in the commutator with $v^{(1)}$ in (68) because $\rho^{(2)}_{F,\kappa}(1;\vec{k}\,)$ is already contained in (66). It is clear that (67) and (68) should hold also for each meson contribution separately.

For the explicit evaluation of the commutators of the potential with the one-body charge density, we first note for a one-body operator

$$a(1;\vec{k},\vec{q},\vec{Q}) = a(\vec{k},\vec{Q})_1\delta(\frac{\vec{k}}{2}-\vec{q}) + (1\leftrightarrow 2) \qquad (69)$$

the general relation

$$\langle\vec{p}\,'\,|\Big[v_B\,,a(1;\vec{k}\,)\Big]|\vec{p}\,\rangle = v_B(-\vec{q}_2,\vec{Q}+\frac{1}{2}\vec{k}\,)\,a(\vec{k},\vec{Q}+\vec{q}_2\,)_1 - a(\vec{k},\vec{Q}-\vec{q}_2\,)_1 v_B(-\vec{q}_2,\vec{Q}-\frac{1}{2}\vec{k}\,)$$
$$+(1\leftrightarrow 2)\,. \qquad (70)$$

Specializing now to the charge density operator as given in (47) and (55) and using the isospin dependence as introduced in (A.8)-(A.10), we find the following relations

$$\langle \vec{p}\,' \,|\, \Big[v_B^{(1)}\,, \rho_F^{(0)}(1;\vec{k}\,) \Big] \,|\vec{p}\,\rangle = -F_{e,1}^{-} \tilde{v}_B^{(1)}(\vec{q}_2\,) + (1 \leftrightarrow 2)\,, \tag{71}$$

$$\begin{aligned}\langle \vec{p}\,' \,|\, \Big[v_B^{(1)}\,, \rho_{F,e}^{(2)}(1;\vec{k}\,) \Big] \,|\vec{p}\,\rangle = & -i\frac{F_{e,1}^{+}}{32m^2}\Big(\Big[\tilde{v}_B^{(1)}(\vec{q}_2\,),\, (\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\Big] \\ & +\Big\{\tilde{v}_B^{(1)}(\vec{q}_2\,),\, (\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}_2\times\vec{k})\Big\}\Big) \\ & +\frac{F_{e,1}^{-}}{32m^2}\Big(4\vec{k}^{\,2}\tilde{v}_B^{(1)}(\vec{q}_2\,) + i\Big\{\tilde{v}_B^{(1)}(\vec{q}_2\,),\, (\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\Big\} \\ & +i\Big[\tilde{v}_B^{(1)}(\vec{q}_2\,),\, (\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}_2\times\vec{k})\Big]\Big) + (1\leftrightarrow 2)\,. \end{aligned} \tag{72}$$

$$\begin{aligned}\langle \vec{p}\,' \,|\, \Big[v_B^{(3)}\,, \rho_F^{(0)}(1;\vec{k}\,) \Big] \,|\vec{p}\,\rangle = & \frac{F_{e,1}^{+}}{2}\Big(\tilde{v}_B^{(3)}(-\vec{q}_2,\vec{Q}+\frac{1}{2}\vec{k}\,) - \tilde{v}_B^{(3)}(-\vec{q}_2,\vec{Q}-\frac{1}{2}\vec{k}\,)\Big) \\ & -\frac{F_{e,1}^{-}}{2}\Big(\tilde{v}_B^{(3)}(-\vec{q}_2,\vec{Q}+\frac{1}{2}\vec{k}\,) + \tilde{v}_B^{(3)}(-\vec{q}_2,\vec{Q}-\frac{1}{2}\vec{k}\,)\Big) \\ & +(1\leftrightarrow 2)\,, \end{aligned} \tag{73}$$

These relations will be useful for checking the continuity condition (68) for the various meson contributions. Since the interaction currents and the commutators in (68) separate into "+" and "−" parts with respect to $F^{\pm}$ and $G^{\pm}$, we can check the continuity condition separately for these parts.

There are two interesting properties of the relativistic part of the continuity equation that are demonstrated explicitly below for particular meson exchanges. First, since we use the representation in which there is no intrinsic retardation potential ($\nu = 1/2$), the retardation part of the current has to satisfy

$$\begin{aligned}\vec{k}\cdot\vec{\jmath}_F^{\,(3)}(2;\mathrm{ret};\vec{k},\vec{q},\vec{Q}) &= \frac{(\vec{q}\cdot\vec{Q})}{m}\,\rho_F^{(2)}(2;\mathrm{ret};\vec{k},\vec{q},\vec{Q}) \\ &= \langle \vec{p}\,' \,|\, \Big[t^{(1)}\,, \rho_F^{(2)}(2;\mathrm{ret};\vec{k}\,) \Big] \,|\vec{p}\,\rangle\,. \end{aligned} \tag{74}$$

This is possible only when the boost contributions from χ_r are included. Consequently, if retardation is considered, it makes little sense to take the corresponding FW operators neglecting at the same time the effects of the wave function boost. Another consequence is that in (68) we need to consider the "pro" and "mes" MEC contributions as defined in Appendix A only.

Second, between the commutators $\Big[v^{(1)}\,,\, \rho^{(2)}_{F,e}(1;\vec k\,)\Big]$ and $\Big[v^{(3)}_{LS}\,,\, \rho^{(0)}_{F}(1;\vec k\,)\Big]$ there are cancellations, where $v^{(3)}_{LS}$ is the spin-orbit part of the potential. Recall, that also in the relativistic charge density $\rho^{(2)}_{F,e}(1;\vec k\,)$ a cancellation occurs between the spin-orbit FW term and the χ_σ term. This suggests that for a proper description of the relativistic intrinsic MECs the inclusion of the spin part of the boost is also important.

Now we will derive the detailed expressions for the intrinsic meson exchange current operators. The FW-operators are listed in Appendix A. They have to be evaluated in the Breit frame using (A.5)-(A.7). In order to keep track of the various pieces contributing to the intrinsic operators, we give in Tables I and II a survey on the nonvanishing terms with reference to the corresponding equations where the explicit expressions are given. In the case of nonrelativistic contributions, the intrinsic operators are given by the FW-ones.

B. Scalar meson exchange

The intrinsic potential contributions including leading relativistic order following from the exchange of a single scalar meson are

$$\tilde v^{(1)}_S(\vec q\,) = -\frac{g_S^2}{(2\pi)^3}\ \tilde\Delta_S(\vec q^{\,2})\,, \tag{75}$$

$$\begin{aligned}\tilde v^{(3)}_S(\vec q,\vec Q) &= -\frac{1}{8m^2}\,\tilde v^{(1)}_S(\vec q\,)\Big(2\vec Q^{\,2} + i(\vec\sigma_1+\vec\sigma_2)\cdot(\vec q\times\vec Q)\Big)\\ &= \tilde v^{(3)}_{S,Q}(\vec q,\vec Q) + \tilde v^{(3)}_{S,LS}(\vec q,\vec Q)\,.\end{aligned} \tag{76}$$

Here, $\tilde\Delta_S(\vec q^{\,2})$ denotes the meson propagator including a hadronic form factor (see Appendix A). The last term in (76) is the spin-orbit potential. In some OBE potentials the following replacement is done in the first term of (76)

$$\vec Q^{\,2} = \vec Q^{\,2} + \vec q^{\,2} - \vec q^{\,2} \to \vec Q^{\,2} + \vec q^{\,2} + m_S^2\,, \tag{77}$$

since $\tilde v^{(1)}_S(\vec q\,)(\vec Q^{\,2} + \vec q^{\,2})/2$ corresponds to the intrinsic anticommutator $\{\hat{\vec p}^{\,2},\, v^{(1)}_S\}$.

Turning now to the corresponding exchange currents, we will start from its "+" part (see Appendix A). Since in this case there are no nonrelativistic contributions neither to charge

nor to current densities, the intrinsic operators are simply given by the FW ones taken in the Breit frame. In all following expressions we have kept the notation $\vec{q}_2 = \frac{1}{2}\vec{k} - \vec{q}$. Hence, we get from (A.25)-(A.27)

$$\vec{\jmath}_F^{\,(3)}(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})^+_S = -F^+_{e,1}\frac{1}{4m^2}\tilde{v}^{(1)}_S(\vec{q}_2\,)(\vec{Q}+i\vec{\sigma}_1\times\vec{k}) + (1\leftrightarrow 2)\,, \tag{78}$$

$$\rho^{(2)}_F(2;\mathrm{ret};\vec{k},\vec{q},\vec{Q})^+_S = F^+_{e,1}\,\frac{g_S^2}{(2\pi)^3\,8m}\,(\vec{k}\cdot\vec{q}_2)\,\tilde{\Delta}'_S(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,, \tag{79}$$

$$\begin{aligned}\vec{\jmath}_F^{\,(3)}(2;\mathrm{ret};\vec{k},\vec{q},\vec{Q})^+_S = {} & \frac{g_S^2}{(2\pi)^3\,16m^2}\Big\{F^+_{e,1}\,\Big[(\vec{k}\cdot\vec{q}_2)\,\vec{Q} - 2(\vec{Q}\cdot\vec{q}_2)\,\vec{q}_2\Big] \\ & + iG^+_{M,1}\,(\vec{k}\cdot\vec{q}_2)\,\vec{\sigma}_1\times\vec{k}\Big\}\,\tilde{\Delta}'_S(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,,\end{aligned} \tag{80}$$

where $\tilde{\Delta}'_S(\vec{q}_2^{\,2})$ is defined in (A.20). Note that for the exchange $(1\leftrightarrow 2)$ one has $\vec{q}_1 = \frac{1}{2}\vec{k} + \vec{q}$.

As next we will look at the consequences of the continuity equation. The retardation charge and current obviously satisfy (74). Thus remains the divergence of the "pro" current which is

$$\vec{k}\cdot\vec{\jmath}_F^{\,(3)}(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})^+_S = -\frac{F^+_{e,1}}{4m^2}\,(\vec{k}\cdot\vec{Q})\,\tilde{v}^{(1)}_S(\vec{q}_2\,)\, + (1\leftrightarrow 2)\,. \tag{81}$$

This indeed is the sum of the commutators on the r.h.s. of (68) because one finds directly from (71) through (73) with (75) and (76)

$$\langle\vec{p}^{\,\prime}\,|\Big[v^{(1)}_S\,,\,\rho^{(0)}_F(1;\vec{k}\,)\Big]^+|\vec{p}\,\rangle = 0\,, \tag{82}$$

$$\langle\vec{p}^{\,\prime}\,|\Big[v^{(1)}_S\,,\,\rho^{(2)}_{F,e}(1;\vec{k}\,)\Big]^+|\vec{p}\,\rangle \;= -i\frac{F^+_{e,1}}{16m^2}\,\tilde{v}^{(1)}_S(\vec{q}_2\,)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}_2\times\vec{k}) + (1\leftrightarrow 2)\,, \tag{83}$$

$$\begin{aligned}\langle\vec{p}^{\,\prime}\,|\Big[v^{(3)}_S\,,\,\rho^{(0)}_F(1;\vec{k}\,)\Big]^+|\vec{p}\,\rangle \;= {} & -\frac{F^+_{e,1}}{16m^2}\,\tilde{v}^{(1)}_S(\vec{q}_2\,)\Big(4(\vec{k}\cdot\vec{Q}) - i(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}_2\times\vec{k})\Big) \\ & +(1\leftrightarrow 2)\,,\end{aligned} \tag{84}$$

where the superscript "+", referring to the isospin dependence, indicates that only the "+" part of the commutator is retained. Clearly, the resulting current and the continuity equation it satisfies are completely different from those one would obtain, e.g., by a minimal replacement in the LS potential neglecting further relativistic effects.

Let us now consider the "−" part of the operators. There is one nonrelativistic term in (A.28) which we will not list here again. The relativistic contributions to the charge and

current densities follow from (A.29)-(A.33) and the boost and separation ones are listed in Appendix B.

After summing up these various terms one obtains finally for the relativistic intrinsic operators

$$
\begin{aligned}
\rho_F^{(2)}(2;\text{mes};\vec{k},\vec{q},\vec{Q})_S^- &= F_{e,1}^- \frac{g_S^2}{(2\pi)^3\,2m}\,(\vec{k}\cdot\vec{Q})\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,, &(85)\\
\rho_F^{(2)}(2;\text{ret};\vec{k},\vec{q},\vec{Q})_S^- &= F_{e,1}^- \frac{g_S^2}{(2\pi)^3\,4m}\,(\vec{q}_2\cdot\vec{Q})\,\tilde{\Delta}_S'(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,. &(86)\\
\vec{\jmath}_F^{\,(3)}(2;\text{pro};\vec{k},\vec{q},\vec{Q})_S^- &= -F_{e,1}^- \frac{g_S^2}{(2\pi)^3\,4m^2}\,(\frac{7}{8}\vec{k} + i\vec{\sigma}_1\times\vec{Q})\;\;\tilde{\Delta}_S(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\\
&= -F_{e,1}^- \frac{g_S^2}{(2\pi)^3\,4m^2}\,(\frac{3}{4}\vec{k} + i\vec{\sigma}_1\times\vec{Q})\;\;\tilde{\Delta}_S(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\\
&\quad -\frac{\vec{k}}{32m^2}\,\vec{k}\cdot\vec{\jmath}_F^{\,(1)}(2;\text{mes};\vec{k},\vec{q})_S^-\,, &(87)\\
\vec{\jmath}_F^{\,(3)}(2;\text{mes};\vec{k},\vec{q},\vec{Q})_S^- &= -F_{e,1}^- \frac{g_S^2}{(2\pi)^3\,4m^2} f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,\vec{q}\,\Big[2\vec{Q}^{\,2} - i(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{q}\,)\\
&\quad -i(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k}) - 2\frac{(\vec{k}\cdot\vec{Q})}{(\vec{k}\cdot\vec{q}\,)}\,(\vec{q}\cdot\vec{Q})\,\Big]\,, &(88)\\
\vec{\jmath}_F^{\,(3)}(2;\text{ret};\vec{k},\vec{q},\vec{Q})_S^- &= -\frac{g_S^2}{(2\pi)^3\,8m^2}\Big[\frac{F_{e,1}^-}{(\vec{k}\cdot\vec{q}\,)}\,\Big(\frac{1}{4}(\vec{k}\cdot\vec{q}_2)\,\vec{k}\times(\vec{k}\times\vec{q}\,)\\
&\quad +(\vec{Q}\cdot\vec{q}_2)\,\Big[\vec{k}\times(\vec{q}\times\vec{Q}\,) - 2\vec{q}\,(\vec{q}\cdot\vec{Q})\Big]\Big)\\
&\quad -iG_{M,1}^-\,(\vec{Q}\cdot\vec{q}_2)\,\vec{\sigma}_1\times\vec{k}\,\Big]\,\tilde{\Delta}_S'(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,. &(89)
\end{aligned}
$$

The nonrelativistic current satisfies

$$
\vec{k}\cdot\vec{\jmath}_F^{\,(1)}(2;\text{mes};\vec{k},\vec{q}\,)_S^- = -F_{e,1}^-\,\tilde{v}_S^{(1)}(\vec{q}_2\,) + (1\leftrightarrow 2)\,, \tag{90}
$$

which in conjunction with (71) is in agreement with (67). Obviously, the retardation charge and current densities satisfy (74). For the remaining divergence of the relativistic “pro” and “mes” currents, one finds from the above expressions

$$
\begin{aligned}
&\vec{k}\cdot\Big(\vec{\jmath}_F^{\,(3)}(2;\text{pro};\vec{k},\vec{q},\vec{Q})_S^- + \vec{\jmath}_F^{\,(3)}(2;\text{mes};\vec{k},\vec{q},\vec{Q})_S^-\Big) =\\
&\qquad \frac{F_{e,1}^-}{8m^2}\tilde{v}_S^{(1)}(\vec{q}_2\,)\Big[2\vec{Q}^{\,2} + \frac{3}{2}\vec{k}^{\,2} + i\,(\vec{\sigma}_1+\vec{\sigma}_2)\times\vec{Q}\cdot(\vec{k}-\vec{q}\,)\Big] + (1\leftrightarrow 2)\\
&\qquad +F_{e,1}^-\frac{g_S^2}{(2\pi)^3\,2m^2}\,(\vec{q}\cdot\vec{Q})(\vec{k}\cdot\vec{Q})\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2}) - \frac{\vec{k}^{\,2}}{32m^2}\,\vec{k}\cdot\vec{\jmath}_F^{\,(1)}(2;\text{mes};\vec{k},\vec{q}\,)_S^-\,.
\end{aligned}\tag{91}
$$

The first term on the r.h.s. is equal to the sum of the following two commutators

$$\langle \vec{p}\,' \,|\Big[v_S^{(1)}\,,\, \rho_{F,e}^{(2)}(1;\vec{k}\,)\Big]^- |\vec{p}\,\rangle = \frac{F_{e,1}^-}{16m^2}\tilde{v}_S^{(1)}(\vec{q}_2\,)\,\Big[2\vec{k}^{\,2} + i\,(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k}\,)\Big] + (1\leftrightarrow 2)\,, \tag{92}$$

$$\langle \vec{p}\,' \,|\Big[v_S^{(3)}\,,\, \rho_{F}^{(0)}(1;\vec{k}\,)\Big]^- |\vec{p}\,\rangle = \frac{F_{e,1}^-}{8m^2}\tilde{v}_S^{(1)}(\vec{q}_2\,)\,\Big[2\vec{Q}^{\,2} + \frac{1}{2}\vec{k}^{\,2} + i\,(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot(\vec{Q}\times\vec{q}_2\,)\Big] + (1\leftrightarrow 2)\,, \tag{93}$$

while the next term is just the commutator of $t^{(1)}$ with the mesonic density (85) according to (63). The last term in (91) is the recoil current contribution in (68). This completes the verification of the "$-$" part of the continuity equation.

Finally, let us shortly describe how one can obtain the conserved current if the replacement (77) is made in the relativistic part of the NN potential. First of all, it follows from (73) that the "+" part of the continuity equation is not affected. For the "$-$" part, an additional term arises in the commutator of the nonrelativistic charge density with the potential, namely

$$-\,F_{e,1}^-\frac{g_S^2}{(2\pi)^3\,4m^2}(m_S^2 + \vec{q}_2^{\,2})\;\tilde{\Delta}_S(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,. \tag{94}$$

Notice, that due to the nucleon exchange term this additional contribution vanishes in the absence of strong form factors. This suggests the following prescription to handle the more general case with form factors: (i) Rearrange the different terms of the current without form factors, i.e., shift some part from the "mes" to the "pro" component, so that its divergence explicitly contains the additional piece (94), (ii) Introduce then the strong form factors into the modified potential and the rearranged current. Obviously, this procedure is not unique, but there is no other more consistent way of solving this problem.

The resulting modified current is a little more complicated for scalar meson exchange, but for vector meson exchange it simplifies significantly. Let us consider the vector

$$-\,\frac{1}{2}(\vec{q}_1 - \vec{q}_2)\Big[\;\tilde{\Delta}_S(\vec{q}_2^{\,2}) + (1\leftrightarrow 2) - (2m_S^2 + \vec{q}_1^{\,2} + \vec{q}_2^{\,2})\;\tilde{\Delta}_S(\vec{q}_1^{\,2})\;\tilde{\Delta}_S(\vec{q}_2^{\,2})\Big]\,, \tag{95}$$

which vanishes if the strong form factors are disregarded and whose divergence is just

$$(m_S^2 + \vec{q}_2^{\,2})\;\tilde{\Delta}_S(\vec{q}_2^{\,2}) - (1\leftrightarrow 2)\,. \tag{96}$$

Multiplying (95) with $-F^-_{e,1}\frac{g_S^2}{(2\pi)^3\,4m^2}$ and adding it to the intrinsic current above, one gets a new conserved current for the modified potential. For the currents of this section this means the following replacements in (87) $\frac{3}{4}\vec{k} \to (\frac{3}{4}\vec{k} - \vec{q}\,)$ and in (88) $\vec{Q}^{\,2} \to (\vec{Q}^{\,2} + m_S^2 + \vec{q}^{\,2} + \frac{1}{4}\vec{k}^{\,2})$.

C. Vector meson exchange

The exchange of a vector meson contributes to the potential by

$$\tilde{v}_V^{(1)}(\vec{q}\,) = \frac{g_V^2}{(2\pi)^3}\;\tilde{\Delta}_V(\vec{q}^{\,2})\,, \tag{97}$$

$$\begin{aligned}\tilde{v}_V^{(3)}(\vec{q},\vec{Q}) &= -\frac{1}{4m^2}\,\tilde{v}_V^{(1)}(\vec{q}\,)\Big[(1+2\kappa_V)\vec{q}^{\,2} + (1+\kappa_V)^2(\vec{\sigma}_1\times\vec{q}\,)\cdot(\vec{\sigma}_2\times\vec{q}\,)\\ &\qquad -\vec{Q}^{\,2} - i(\frac{3}{2}+2\kappa_V)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}\times\vec{Q})\Big]\\ &= \tilde{v}_{V,q}^{(3)}(\vec{q},\vec{Q}) + \tilde{v}_{V,\sigma q}^{(3)}(\vec{q},\vec{Q}) + \tilde{v}_{V,Q}^{(3)}(\vec{q},\vec{Q}) + \tilde{v}_{V,LS}^{(3)}(\vec{q},\vec{Q})\,.\end{aligned} \tag{98}$$

The term $\tilde{v}_{V,\sigma q}^{(3)}(\vec{q},\vec{Q})$ contains a central spin-spin and a tensor part. Again, $\vec{q}^{\,2}$ is often replaced by $-m_V^2$ and then $\tilde{v}_V^{(3)}(\vec{q},\vec{Q})$ may be redefined to be

$$\begin{aligned}\hat{v}_V^{(3)}(\vec{q},\vec{Q}) = \frac{1}{4m^2}\,\tilde{v}_V^{(1)}(\vec{q}\,)\Big\{&(1+2\kappa_V)m_V^2 + (1+\kappa_V)^2\Big[m_V^2(\vec{\sigma}_1\cdot\vec{\sigma}_2) + (\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}\,)\Big]\\ &+\vec{Q}^{\,2} + i(\frac{3}{2}+2\kappa_V)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}\times\vec{Q})\Big\}\,.\end{aligned} \tag{99}$$

Like the nonrelativistic potential $\tilde{v}_V^{(1)}(\vec{q}\,)$, many currents are obtained from those of scalar exchange given in the previous section by the replacements $m_S \to m_V$ and $g_S^2 \to -g_V^2$.

For the "+" part, one gets in this way the retardation current from (79) and (80). In addition, there is the "pro" part of the current which follows from (A.34)

$$\vec{\jmath}_F^{\,(3)}(2;\text{pro};\vec{k},\vec{q},\vec{Q})_V^+ = \frac{F^+_{e,1}}{4m^2}\tilde{v}_V^{(1)}(\vec{q}_2\,)\Big[\vec{Q} - i(1+\kappa_V)(\vec{\sigma}_1+\vec{\sigma}_2)\times\vec{q}_2\Big] + (1\leftrightarrow 2)\,. \tag{100}$$

Its divergence should equal the sum of the following commutators

$$\langle\vec{p}\,'\,|\Big[v_V^{(1)}\,,\,\rho_{F,e}^{(2)}(1)\Big]^+|\vec{p}\,\rangle \;= -i\frac{F^+_{e,1}}{16m^2}\tilde{v}_V^{(1)}(\vec{q}_2\,)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}_2\times\vec{k}\,) + (1\leftrightarrow 2)\,, \tag{101}$$

$$\begin{aligned}\langle\vec{p}\,'\,|\Big[v_V^{(3)}\,,\,\rho_F^{(0)}(1;\vec{k}\,)\Big]^+|\vec{p}\,\rangle = \frac{F^+_{e,1}}{8m^2}\tilde{v}_V^{(1)}(\vec{q}_2\,)\Big[&2(\vec{k}\cdot\vec{Q}) - i(\frac{3}{2}+2\kappa_V)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}_2\times\vec{k}\,)\Big]\\ &+(1\leftrightarrow 2)\,,\end{aligned} \tag{102}$$

which is easy to verify.

For the "−" part the replacements $m_S \to m_V$ and $g_S^2 \to -g_V^2$ yield the nonrelativistic current $\vec{\jmath}_F^{(1)}(2;\text{mes};\vec{k},\vec{q})_V^-$ from (A.28), the retardation charge $\rho_F^{(2)}(2;\text{ret};\vec{k},\vec{q},\vec{Q})_V^-$ and current $\vec{\jmath}_F^{(3)}(2;\text{ret};\vec{k},\vec{q},\vec{Q})_V^-$ from (86) and (89), respectively, and the mesonic charge $\rho_F^{(2)}(2;\text{mes};\vec{k},\vec{q},\vec{Q})_V^-$ from (85). In order to get the remaining currents, one should take (A.35)-(A.38) and add the nonretardation parts of the boost and separation contributions, i.e., (B.2) and (B.1) with the appropriate replacement of the meson parameters. Keeping the "mes-tr" mesonic current separately (see appendix A), we finally obtain

$$
\begin{aligned}
\vec{\jmath}_F^{(3)}(2;\text{pro};\vec{k},\vec{q},\vec{Q})_V^- &= F_{e,1}^- \frac{g_V^2}{(2\pi)^3\,4m^2}\Big[(1+\kappa_V)\vec{q}_2 - \kappa_V\vec{q}_1 - \frac{1}{4}\vec{k} - i(1+2\kappa_V)\vec{\sigma}_1\times\vec{Q} \\
&\quad -(1+\kappa_V)^2\vec{\sigma}_1\times(\vec{\sigma}_2\times\vec{q}_2)\Big]\ \tilde{\Delta}_V(\vec{q}_2^{\,2}) + (1\leftrightarrow 2) \\
&\quad -\frac{\vec{k}}{32m^2}\,\vec{k}\cdot\vec{\jmath}_F^{(1)}(2;\text{mes};\vec{k},\vec{q})_V^-\,, \qquad (103)\\
\vec{\jmath}_F^{(3)}(2;\text{mes};\vec{k},\vec{q},\vec{Q})_V^- &= F_{e,1}^- \frac{g_V^2}{(2\pi)^3\,2m^2} f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,\vec{q}\Big[(1+2\kappa_V)(\vec{q}^{\,2}+\frac{\vec{k}^{\,2}}{4}) - \vec{Q}^{\,2} \\
&\quad -(1+\kappa_V)^2(\vec{\sigma}_1\times\vec{q}_1)\cdot(\vec{\sigma}_2\times\vec{q}_2) + i(\frac{3}{2}+2\kappa_V)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{q}) \\
&\quad +i(\frac{1}{2}+\kappa_V)(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k}) - \frac{(\vec{k}\cdot\vec{Q})}{(\vec{k}\cdot\vec{q})}(\vec{q}\cdot\vec{Q})\Big]\,, \qquad (104)\\
\rho_F^{(2)}(2;\text{mes-tr};\vec{k},\vec{q},\vec{Q})_V^- &= F_{e,1}^- \frac{g_V^2}{(2\pi)^3\,2m} f(\vec{q}_1^{\,2},\vec{q}_2^{\,2}) \\
&\quad \Big[2(\vec{k}\cdot\vec{Q}) + i(1+\kappa_V)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}\times\vec{k})\Big]\,, \qquad (105)\\
\vec{\jmath}_F^{(3)}(2;\text{mes-tr};\vec{k},\vec{q},\vec{Q})_V^- &= F_{e,1}^- \frac{g_V^2}{(2\pi)^3\,4m^2} f(\vec{q}_1^{\,2},\vec{q}_2^{\,2}) \\
&\quad \Big\{2(\vec{q}\cdot\vec{Q})\Big[2\vec{Q} + i(1+\kappa_V)\Big((\vec{\sigma}_1+\vec{\sigma}_2)\times\vec{q} + \frac{1}{2}(\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{k}\Big)\Big] \\
&\quad +\vec{k}\times\Big[(1+\kappa_V)^2\,(\vec{\sigma}_1\times\vec{q}_1)\times(\vec{\sigma}_2\times\vec{q}_2) \\
&\quad +i(1+\kappa_V)\Big((\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{q} + \frac{1}{2}(\vec{\sigma}_1+\vec{\sigma}_2)\times\vec{k}\Big)\times\vec{Q}\Big]\Big\}\,. \qquad (106)
\end{aligned}
$$

Let us now examine the continuity equation for the "−" part of the current. Its nonrelativistic and retardation parts are satisfied exactly as in the case of scalar meson exchange. Since also the transverse mesonic current satisfies

$$
\vec{k}\cdot\vec{\jmath}_F^{(3)}(2;\text{mes-tr};\vec{k},\vec{q},\vec{Q})_V^- = \frac{(\vec{q}\cdot\vec{Q})}{m}\,\rho_F^{(2)}(2;\text{mes-tr};\vec{k},\vec{q},\vec{Q})_V^-\,, \qquad (107)
$$

we are left with the divergence of the relativistic "pro" and "mes" currents for which one finds

$$\vec{k}\cdot\Big(\vec{\jmath}^{\,(3)}_F(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})^-_V+\vec{\jmath}^{\,(3)}_F(2;\mathrm{mes};\vec{k},\vec{q},\vec{Q})^-_V\Big)=$$
$$\frac{F^-_{e,1}}{4m^2}\tilde{v}^{(1)}_V(\vec{q}_2\,)\Big(\frac{1}{4}\vec{k}^{\,2}-\vec{Q}^{\,2}+(1+2\kappa_V)\vec{q}^{\,2}_2+(1+\kappa_V)^2(\vec{\sigma}_1\times\vec{q}_2)\cdot(\vec{\sigma}_2\times\vec{q}_2)$$
$$+i\Big(\frac{3}{2}+2\kappa_V\Big)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{q}\,)-i\Big(\frac{1}{2}+\kappa_V\Big)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\Big)+(1\leftrightarrow 2)$$
$$-F^-_{e,1}\frac{g^2_V}{(2\pi)^3\,2m^2}\,(\vec{q}\cdot\vec{Q})(\vec{k}\cdot\vec{Q})\,f(\vec{q}^{\,2}_1,\vec{q}^{\,2}_2)-\frac{\vec{k}^{\,2}}{32m^2}\,\vec{k}\cdot\vec{\jmath}^{\,(1)}_F(2;\mathrm{mes};\vec{k},\vec{q}\,)^-_V\,. \tag{108}$$

Again, the first term on the r.h.s. is equal to the sum of the following two commutators

$$\langle\vec{p}^{\,\prime}\,|\Big[v^{(1)}_V\,,\,\rho^{(2)}_{F,e}(1;\vec{k}\,)\Big]^-|\vec{p}\,\rangle=\frac{F^-_{e,1}}{16m^2}\,\tilde{v}^{(1)}_V(\vec{q}_2\,)\Big(2\vec{k}^{\,2}+i(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\Big)+(1\leftrightarrow 2)\,, \tag{109}$$

$$\langle\vec{p}^{\,\prime}\,|\Big[v^{(3)}_V\,,\,\rho^{(0)}_{F}(1;\vec{k}\,)\Big]^-|\vec{p}\,\rangle=\frac{F^-_{e,1}}{4m^2}\,\tilde{v}^{(1)}_V(\vec{q}_2\,)\Big((1+2\kappa_V)\,\vec{q}^{\,2}_2+(1+\kappa_V)^2\,(\vec{\sigma}_1\times\vec{q}_2)\cdot(\vec{\sigma}_2\times\vec{q}_2)$$
$$-\vec{Q}^{\,2}-\frac{1}{4}\vec{k}^{\,2}-i(\frac{3}{2}+2\kappa_V)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{q}_2)\Big)+(1\leftrightarrow 2)\,, \tag{110}$$

while the next term is just the commutator of $t^{(1)}$ with the mesonic density and the last term in (108) is the recoil current contribution in (68). Thus the continuity condition is satisfied also for vector meson exchange.

Finally, in order to obtain the conserved currents for the modified potential (99), one can use again the same procedure as in the previous section. Namely, switching off the strong form factors for a moment and using in the mesonic current the following identities

$$(\vec{q}_1\cdot\vec{q}_2)=\frac{\vec{k}^{\,2}}{2}-(\vec{q}^{\,2}_1+\vec{q}^{\,2}_2)\,, \tag{111}$$

$$\vec{q}^{\,2}_1+\vec{q}^{\,2}_2=-2m^2_V+(m^2_V+\vec{q}_1)+(m^2_V+\vec{q}_2)\,, \tag{112}$$

one gets the modified intrinsic currents

$$\vec{\jmath}^{\,(3)}_F(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})^-_V=F^-_{e,1}\frac{g^2_V}{(2\pi)^3\,4m^2}\,\Big(\frac{1}{4}\vec{k}+(1+\kappa_V)^2\Big(\frac{\vec{k}}{2}(\vec{\sigma}_1\cdot\vec{\sigma}_2)-\vec{\sigma}_2(\vec{\sigma}_1\cdot\vec{q}_2)\Big)$$
$$-i(1+2\kappa_V)\vec{\sigma}_1\times\vec{Q}\Big)\;\tilde{\Delta}_V(\vec{q}^{\,2}_2)+(1\leftrightarrow 2)$$
$$-\frac{\vec{k}}{32m^2}\,\vec{k}\cdot\vec{\jmath}^{\,(1)}_F(2;\mathrm{mes};\vec{k},\vec{q}\,)^-_V\,, \tag{113}$$

$$\vec{\jmath}^{\,(3)}_F(2;\mathrm{mes};\vec{k},\vec{q},\vec{Q})^-_V=F^-_{e,1}\,\frac{g^2_V}{(2\pi)^3\,2m^2}f(\vec{q}^{\,2}_1,\vec{q}^{\,2}_2)\,\vec{q}\,\Big[-(1+2\kappa_V)m^2_V-\vec{Q}^{\,2}$$

$$+i(\frac{3}{2}+2\kappa_V)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{q}\,)+i(\frac{1}{2}+\kappa_V)(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})$$
$$+(1+\kappa_V)^2\Big((\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_1)-(m_V^2+\frac{\vec{k}^{\,2}}{2})(\vec{\sigma}_1\cdot\vec{\sigma}_2)\Big)$$
$$-\frac{(\vec{k}\cdot\vec{Q})}{(\vec{k}\cdot\vec{q})}\,(\vec{q}\cdot\vec{Q})\Big]. \tag{114}$$

Unlike for the scalar exchange, the modified current is in this case somewhat simpler than the original one.

D. Pseudoscalar meson exchange

For pseudoscalar exchange one faces the problem of different, unitarily equivalent representations which can be characterized by a parameter $\tilde{\mu}$ (see Appendix A). The simplest form of the pseudoscalar exchange potential corresponds to the representation with $\tilde{\mu}=0$. In this case, only one simple relativistic contribution appears besides the nonrelativistic potential

$$\tilde{v}_{PS}^{(1)}(\vec{q}\,)=-\frac{g_{PS}^2}{(2\pi)^3\,4m^2}\,(\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}\,)\;\tilde{\Delta}_{PS}(\vec{q}^{\,2})\,, \tag{115}$$
$$\tilde{v}_{PS}^{(3)}(\vec{q},\vec{Q})=\frac{g_{PS}^2}{(2\pi)^3\,16m^4}\,(\vec{Q}^{\,2}+\vec{q}^{\,2})\,(\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}\,)\;\tilde{\Delta}_{PS}(\vec{q}^{\,2})\,. \tag{116}$$

Since the operator form of the last term is $-\{\vec{p}^{\,2},\,v_{PS}^{(1)}\}$, its inclusion into the Lippmann-Schwinger equation causes numerical instabilities [13]. Therefore, most realistic NN potentials disregard any relativistic correction to the OPEP.

Let us first consider the "+" part of the e.m. operators. The intrinsic currents follow from (A.49) and (A.50) in conjunction with the expressions of (A.53)-(A.56), (A.47)-(A.48) and (B.16)

$$\rho_F^{(2)}(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})_{PS}^{+}=F_{e,1}^{+}\,\frac{g_{PS}^2}{(2\pi)^3\,32m^3}\Big[3(\vec{\sigma}_1\cdot\vec{k}\,)(\vec{\sigma}_2\cdot\vec{q}_2)-\frac{1}{2}\,(\vec{k}\times\vec{q}_2)\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)\Big]\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})$$
$$+(1\leftrightarrow 2)\,, \tag{117}$$
$$\vec{\jmath}_F^{\,(3)}(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})_{PS}^{+}=\frac{g_{PS}^2}{(2\pi)^3\,16m^4}\Big\{F_{e,1}^{+}\Big[\vec{Q}\Big(\vec{\sigma}_1\cdot(\vec{k}-\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}_2)+\frac{1}{8}\,\Sigma^{(+)}(\vec{q}_2,\,\vec{k})\Big)$$
$$-\vec{\sigma}_1(\vec{q}_2\cdot\vec{Q})(\vec{\sigma}_2\cdot\vec{q}_2)-i\vec{k}\times\vec{q}_2(\vec{\sigma}_2\cdot\vec{q}_2)-\frac{1}{4}\vec{q}_2\,\Sigma^{(+)}(\vec{q}_2,\,\vec{Q})\Big]$$

$$+\frac{1}{4}G^{+}_{M,1}\,\vec{k}\times\Big[3\vec{\sigma}_1\times\vec{Q}\,(\vec{\sigma}_2\cdot\vec{q}_2)-\vec{\sigma}_1\times\vec{q}_2\,(\vec{\sigma}_2\cdot\vec{Q})$$

$$-\frac{i}{2}\vec{q}_2(\vec{\sigma}_2\cdot\vec{k}\,)\Big]\,\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{118}$$

$$\rho^{(2)}_{F}(2;\mathrm{ret};\vec{k},\vec{q},\vec{Q})^{+}_{PS}=F^{+}_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^3}\,(\vec{k}\cdot\vec{q}_2)(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2)\,\tilde{\Delta}'_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{119}$$

$$\vec{\jmath}^{\,(3)}_{F}(2;\mathrm{ret};\vec{k},\vec{q},\vec{Q})^{+}_{PS}=\frac{g^2_{PS}}{(2\pi)^3\,64m^4}\Big\{F^{+}_{e,1}\,\Big[2\vec{q}_2\,(\vec{Q}\cdot\vec{q}\,)+\vec{k}\times(\vec{Q}\times\vec{q}_2)\Big](\vec{\sigma}_1\cdot\vec{q}_2)$$

$$-G^{+}_{M,1}\,\vec{k}\times\Big[2\vec{\sigma}_1\times\vec{q}_2(\vec{Q}\cdot\vec{q}_2)+i\vec{q}_2(\vec{k}\cdot\vec{q}_2)\Big]\Big\}(\vec{\sigma}_2\cdot\vec{q}_2)\,\tilde{\Delta}'_{PS}(\vec{q}_2^{\,2})$$

$$+(1\leftrightarrow 2)\,, \tag{120}$$

where $\Sigma^{(\pm)}(\vec{a},\,\vec{b})$ is defined in (A.42). With respect to the continuity condition (68), one notes that again the retardation operators satisfy (74), while for the divergence of the "pro" current one gets

$$\vec{k}\cdot\vec{\jmath}^{\,(3)}_{F}(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})^{+}_{PS}=F^{+}_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,16m^4}\,\Big\{\Big[(\vec{k}\cdot\vec{Q})(\vec{\sigma}_1\cdot\vec{q}_2)+(\vec{q}\cdot\vec{Q})(\vec{\sigma}_1\cdot\vec{k})\Big](\vec{\sigma}_2\cdot\vec{q}_2)$$

$$-\frac{1}{4}(\vec{k}\cdot\vec{q}_2)\,\Sigma^{(+)}(\vec{q}_2,\,\vec{Q})+\frac{1}{8}(\vec{k}\cdot\vec{Q})\,\Sigma^{(+)}(\vec{q}_2,\,\vec{k})\Big\}\,\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})$$

$$+(1\leftrightarrow 2)\,. \tag{121}$$

Explicit evaluation of the commutators on the r.h.s. of (68) leads to

$$\langle\vec{p}\,'\,|\Big[v^{(1)}_{PS}\,,\,\rho^{(0)}(1;\vec{k}\,)\Big]^{+}|\vec{p}\,\rangle=0\,, \tag{122}$$

$$\langle\vec{p}\,'\,|\Big[t^{(1)}\,,\,\rho^{(2)}_{F}(2;\mathrm{pro};\vec{k}\,)^{+}_{\mathrm{PS}}\Big]|\vec{p}\,\rangle=F^{+}_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,64m^4}\,(\vec{q}\cdot\vec{Q})\Big[5(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\vec{q}_2)$$

$$+(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{k})\Big]\,\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{123}$$

$$\langle\vec{p}\,'\,|\Big[v^{(1)}_{PS}\,,\,\rho^{(2)}_{F,e}(1;\vec{k}\,)\Big]^{+}|\vec{p}\,\rangle=F^{+}_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,64m^4}\Big[(\vec{Q}\cdot\vec{q}_2)\Sigma^{(+)}(\vec{q}_2,\,\vec{k})$$

$$-(\vec{k}\cdot\vec{q}_2)\Sigma^{(+)}(\vec{q}_2,\,\vec{Q})\Big]\,\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{124}$$

$$\langle\vec{p}\,'\,|\Big[v^{(3)}_{PS},\,\rho^{(0)}(1;\vec{k}\,)\Big]^{+}|\vec{p}\,\rangle=F^{+}_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,16m^4}\,(\vec{k}\cdot\vec{Q})(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2)\,\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})$$

$$+(1\leftrightarrow 2)\,, \tag{125}$$

and their appropriate sum equals (121).

With respect to the "−" currents, we do not repeat the expressions for the nonrelativistic currents (A.57)-(A.58), unchanged they define the corresponding nonrelativistic intrinsic

ones. The FW-currents for $\tilde{\mu} = 0$ follow from (A.49)-(A.50). Let us first collect the charge densities. There are no additional contributions to them from the χ_σ and χ_r boost commutators since there is no nonrelativistic exchange charge operator, and the "$-$" part of the interaction dependent boost χ_V of (B.15) disappears in the Breit frame. Thus one finds

$$\rho_F^{(2)}(2;\text{pro};\vec{k},\vec{q},\vec{Q})^-_{PS} = F^-_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,32m^3}\Sigma^{(+)}(\vec{q}_2,\,\vec{Q})\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,, \tag{126}$$

$$\rho_F^{(2)}(2;\text{mes};\vec{k},\vec{q},\vec{Q})^-_{PS} = -F^-_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,8m^3}\,(\vec{k}\cdot\vec{Q})\,(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,, \tag{127}$$

$$\rho_F^{(2)}(2;\text{ret};\vec{k},\vec{q},\vec{Q})^-_{PS} = F^-_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,16m^3}\,(\vec{Q}\cdot\vec{q}_2)\,(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2)\;\tilde{\Delta}'_{PS}(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,. \tag{128}$$

The spatial currents are more complicated than those for the scalar and vector meson exchanges, since first of all the nonrelativistic meson nucleon vertex depends on the nucleon spin and, furthermore, there is a nonrelativistic "pro" current generating additional boost and separation contributions, which are listed in Appendix B. The $\tilde{\mu}$-independent FW-current densities are listed in (A.61), (A.63) and (A.65). Combining all terms, we find for the relativistic intrinsic current operators

$$\begin{aligned}
\vec{\jmath}_F^{\,(3)}(2;\text{pro};\vec{k},\vec{q},\vec{Q})^-_{PS} = \frac{g^2_{PS}}{(2\pi)^3\,32m^4}\Big\{&F^-_{e,1}\Big[\vec{\sigma}_1\Big\{\Big(3(\vec{k}\cdot\vec{q}\,) - 4\vec{q}^{\,2} - 2\vec{Q}^{\,2}\Big)(\vec{\sigma}_2\cdot\vec{q}_2)\\
&+(\vec{q}\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}\,) - (\vec{Q}\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{Q}) + \frac{i}{2}(\vec{k}\times\vec{q}\,)\cdot\vec{Q}\Big\}\\
&+\frac{1}{4}\vec{k}\Big\{\Big(7(\vec{\sigma}_1\cdot\vec{q}\,) - 4(\vec{\sigma}_1\cdot\vec{k})\Big)(\vec{\sigma}_2\cdot\vec{q}_2) - \frac{1}{2}(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{k}\,)\Big\}\\
&-\vec{q}\Big\{\vec{\sigma}_1\cdot(\vec{k}+\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2) + \frac{1}{2}(\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{k}\,)\Big\}\\
&-i(2\vec{k}-\vec{q}\,)\times\vec{Q}\,(\vec{\sigma}_2\cdot\vec{q}_2) + \frac{1}{2}\vec{Q}\,\Sigma^{(-)}(\vec{q}_2,\,\vec{Q})\Big]\\
&+\frac{1}{4}G^-_{M,1}\vec{k}\times\Big[\Big(6i\,\vec{Q} + 5\,\vec{k}\times\vec{\sigma}_1\Big)(\vec{\sigma}_2\cdot\vec{q}_2) + \vec{q}_2\times\vec{\sigma}_1\,(\vec{\sigma}_2\cdot\vec{k}\,)\\
&-2i\vec{q}_2\,(\vec{\sigma}_2\cdot\vec{Q})\Big]\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,,
\end{aligned}\tag{129}$$

$$\begin{aligned}
\vec{\jmath}_F^{\,(3)}(2;\text{mes};\vec{k},\vec{q},\vec{Q})^-_{PS} = F^-_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\,&\vec{q}\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\;\Big\{\vec{q}^{\,2}\Big[(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\vec{q}_2) + (\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{k})\Big]\\
&+4\Big(\vec{q}^{\,2} + \vec{Q}^{\,2} - \frac{(\vec{k}\cdot\vec{Q})(\vec{q}\cdot\vec{Q})}{(\vec{k}\cdot\vec{q}\,)}\Big)(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)\\
&+2(\vec{q}_2\cdot\vec{Q})(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{Q}) + 2(\vec{q}_1\cdot\vec{Q})(\vec{\sigma}_1\cdot\vec{Q})(\vec{\sigma}_2\cdot\vec{q}_2)\\
&-i\,(\vec{k}\times\vec{q}\,)\cdot\vec{Q}\,\Big((\vec{\sigma}_1\cdot\vec{q}_1) + (\vec{\sigma}_2\cdot\vec{q}_2)\Big)\Big\}\,,
\end{aligned}\tag{130}$$

$$
\begin{aligned}
\vec{\jmath}_F^{\,(3)}(2;\mathrm{ret};\vec{k},\vec{q},\vec{Q})^-_{PS} = -\frac{g^2_{PS}}{(2\pi)^3\,64m^4}\Big\{F^-_{e,1}\,\Big[\Big(4\,(\vec{q}_2\cdot\vec{Q})^2-(\vec{k}\cdot\vec{q}_2)^2\Big)\Big(\vec{\sigma}_1-\vec{q}\,\frac{(\vec{\sigma}_1\cdot\vec{q}_1)}{(\vec{k}\cdot\vec{q})}\Big)\\
+(\vec{\sigma}_1\cdot\vec{q}_2)\Big(\vec{q}_2\,(\vec{k}\cdot\vec{q}_2)-2\vec{Q}\,(\vec{q}_2\cdot\vec{Q})\Big)\Big]\\
+G^-_{M,1}\vec{k}\times\Big[2i\vec{q}_2\,(\vec{q}_2\cdot\vec{Q})+\vec{\sigma}_1\times\vec{q}_2\,(\vec{k}\cdot\vec{q}_2)\Big]\Big\}(\vec{\sigma}_2\cdot\vec{q}_2)\tilde{\Delta}'_{PS}(\vec{q}_2^{\,2})\\
+(1\leftrightarrow 2)\,. \qquad (131)
\end{aligned}
$$

Finally, with respect to the continuity equation (68), it is easy to see that the "ret"-part satisfies (74). The verification for the remaining current is straightforward but lengthy. Since it is desirable to see in detail the correspondance between the various commutators and the associated currents, we have outlined it in detail in Appendix E.

IV. SUMMARY AND OUTLOOK

In this work we have consistently derived a complete set of intrinsic operators of the e.m. charge and current density operators for one- and two-body interaction currents including leading order relativistic contributions for a one-boson-exchange potential consisting of scalar, pseudoscalar and vector meson exchanges. These operators have to be evaluated between intrinsic rest frame wave functions only, since they include already the effects of the wave function boosts. Furthermore, these operators respect gauge and Lorentz invariance up to the terms of leading relativistic order.

The definition of the intrinsic operators is based upon the general separation of the center-of-mass motion from the intrinsic one as described in [1]. It relies on the general properties of the Poincaré generators in conjunction with a $1/m$-expansion. Although the explicit construction of the operators has been done here in the framework of the extended S-matrix method of [3], other methods are known to give unitarily equivalent results.

These operators will now allow to study relativistic effects in e.m. processes in nuclei in a more systematic and consistent way, at least at not too high energy and momentum transfer, since they rely on the $1/m$-expansion. They will also allow a systematic comparison with covariant approaches [9] currently being applied only for the simplest two-nucleon system.

Confrontation of their results [10] with more conventional ones using the $1/m$-expansion seems to indicate that it might be more important to incorporate relativistic effects into the nuclear currents than into the nuclear dynamics.

At present, there are only a few consistent treatments within the $1/m$-expansion, namely for deuteron photodisintegration using a simplified dynamical model of a pure one-pion-exchange interaction [13] and for electrodisintegration using a realistic interaction model [12,14]. However, additional simplifications have been introduced in [11] by considering only quasifree kinematics where the interaction currents in general are negligible, and in [12] by keeping only local velocity-independent terms. In those approximate treatments, effects due to genuine relativistic components of the intrinsic wave functions have been neglected, too. However, a thorough treatment has also to include the relativistic effects in the internal dynamics. This would in particular imply a readjustment of realistic potential models, because at least some relativistic effects are, implicitly or explicitly, accounted for by fitting some phenomenological parameters of a realistic nucleon-nucleon potential to experimental scattering data, although it is then often inserted into the nonrelativistic Schrödinger equation.

Therefore, the next task will be to include all leading order terms in a realistic calculation for the two-body system, which is not very difficult for the charge operator. However, when one turns to the spatial current, the number of terms increases enormously. But with present day methods, it is not impossible. Particularly interesting will be the study of polarization observables because some of them appear to be very sensitive to relativistic e.m. currents already at rather small momentum transfer [11,19]. In this region it seems reasonable to include them in the framework of the $1/m$-expansion.

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APPENDIX A: FOLDY-WOUTHUYSEN CURRENTS

In this appendix we list the FW one-nucleon and exchange currents as obtained by the extended S-matrix technique of [3]. The equations from [3] are referred to as ATA-(number of eq.). The one-nucleon currents and the one-pion-exchange ones were carefully compared to the results of [13]. A couple of misprints in [3] were found and we correct them below.

We follow the notation of [3] for particle momenta ($i = 1,\, 2$)

$$\vec{q}_i = \vec{p}_i^{\,\prime} - \vec{p}_i \,, \tag{A.1}$$

$$\vec{Q}_i = \vec{p}_i^{\,\prime} + \vec{p}_i \,, \tag{A.2}$$

and for the e.m. form factors

$$\hat{e}_i = \frac{1}{2}\Big(F_1^{is}(k^2) + F_1^{iv}(k^2)\,\tau_i^3\Big)\,, \tag{A.3}$$

$$\hat{\kappa}_i = \frac{1}{2}\Big(F_2^{is}(k^2) + F_2^{iv}(k^2)\,\tau_i^3\Big)\,, \tag{A.4}$$

where τ_i^3 is the third isospin component of the ith nucleon. $F_{1,2}^{is/iv}$ denote the isoscalar and isovector Dirac-Pauli nucleon form factors, respectively. In passing to the Breit frame, one finds for the two nucleon system

$$\vec{q}_{1,2} = \frac{1}{2}\vec{k} \pm \vec{q}\,, \tag{A.5}$$

$$\vec{Q}_{1,2} = \pm\vec{Q}\,, \tag{A.6}$$

with $\vec{q} = \vec{p}^{\,\prime} - \vec{p}$, $\vec{Q} = \vec{p}^{\,\prime} + \vec{p}$ and $\vec{p}^{\,(\prime)} = (\vec{p}_1^{\,(\prime)} - \vec{p}_2^{\,(\prime)})/2$. However, we keep the notation $\vec{q}_{1,2}$ in the expressions for the intrinsic currents instead of (A.5), whenever it simplifies the formulas. The time components of q_i, that describe the energy transferred in the corresponding vertex, are expressed in terms of the on-shell nucleon kinetic energies, i.e., up to the order considered

$$q_{i0} = \frac{1}{2m}(\vec{q}_i \cdot \vec{Q}_i) + \mathcal{O}(m^{-3})\,, \tag{A.7}$$

where m denotes the nucleon mass.

With respect to the meson exchange currents associated with the various mesons of a given OBE potential, it is useful to separate the isospin dependence in the potential contribution. $V_B = T\,\tilde{v}_B$, where $T = (\vec{\tau}_1 \cdot \vec{\tau}_2)$ for an isovector meson, and $T = 1$ for an isoscalar one. Because the isospin dependence of the MECs originate from the e.m. form factors and from the isospin operators of the potential, it is convenient to separate the MECs into pieces proportional to $F^{\pm}_{e/\kappa,i}$ and $G^{\pm}_{M,i}$ as defined by

$$F^{\pm}_{e,i} = \hat{e}_i\,T \,\pm\, T\,\hat{e}_i\,, \tag{A.8}$$

$$F^{\pm}_{\kappa,i} = \hat{\kappa}_i\,T \,\pm\, T\,\hat{\kappa}_i\,, \tag{A.9}$$

$$G^{\pm}_{M,i} = F^{\pm}_{e,i} \,+\, F^{\pm}_{\kappa,i}\,, \tag{A.10}$$

We would like to emphasize that this is *not* the separation into isoscalar and isovector parts. Notice also that $F^{-}_{e/\kappa,i}$ and $G^{-}_{M,i}$ are odd under nucleon interchange and thus vanish for isoscalar exchange.

One-nucleon currents

For completeness we will begin with the well-known expressions for the one-nucleon current in two-nucleon space in an arbitrary frame of reference. To this end we remind the reader at the general definition for the matrix element of an operator $a(\vec{k})$ in two-nucleon momentum space transferring a momentum $\vec{k}$

$$\langle \vec{P}'\,\vec{p}'|a(\vec{k})|\vec{P}\,\vec{p}\rangle = a(\vec{k},\vec{K},\vec{q},\vec{Q})\,\delta(\vec{P}' - \vec{P} - \vec{k})\,, \tag{A.11}$$

with $\vec{K} = \vec{P}' + \vec{P}$. For a one-body operator $a(1;\vec{k})$ one finds

$$\begin{aligned}\langle \vec{P}'\,\vec{p}'|a(1;\vec{k})|\vec{P}\,\vec{p}\rangle &= \langle \vec{p_1}'\,\vec{p_2}'|a(1;\vec{k})|\vec{p_1}\,\vec{p_2}\rangle \\ &= a(1;\vec{k},\vec{Q}_1)_1\delta(\vec{q_2}) + (1 \leftrightarrow 2) \\ &= \Big(a(1;\vec{k},\vec{Q}_1)_1 + (1 \leftrightarrow 2)\Big)\delta(\vec{P}' - \vec{P} - \vec{k})\,.\end{aligned} \tag{A.12}$$

The last line follows from the fact that $a(1;\vec{k},\vec{Q}_1)_1$ contains $\delta(\vec{q}_1-\vec{k})$. Therefore one gets

$$a(1;\vec{k},\vec{K},\vec{q},\vec{Q}) = a(1;\vec{k},\vec{Q}_1)_1 + (1\leftrightarrow 2)\,, \tag{A.13}$$

where $\vec{q}_{1,2}$ is given in (A.5) and $\vec{Q}_{1,2}=\vec{K}/2\pm\vec{Q}$. Thus it is sufficient to list the contribution of nucleon "1", i.e., $a(1;\vec{k},\vec{Q}_1)_1$, since the contribution of the other one is obtained by adding the exchange $(1\leftrightarrow 2)$.

For the one-body current including the leading relativistic order, one has

$$\begin{aligned}
\rho^{(0)}_{FW}(1;\vec{k},\vec{Q}_1)_1 &= \hat{e}_1\delta(\vec{q}_1-\vec{k})\,, && \text{(A.14)}\\
\rho^{(2)}_{FW}(1;\vec{k},\vec{Q}_1)_1 &= -\frac{\hat{e}_1+2\hat{\kappa}_1}{8m^2}\Big(\vec{k}^{\,2}+i\vec{\sigma}_1\cdot(\vec{Q}_1\times\vec{k})\Big)\,\delta(\vec{q}_1-\vec{k})\,, && \text{(A.15)}\\
\vec{\jmath}^{\,(1)}_{FW}(1;\vec{k},\vec{Q}_1)_1 &= \frac{1}{2m}\Big(\hat{e}_1\vec{Q}_1+i(\hat{e}_1+\hat{\kappa}_1)\,\vec{\sigma}_1\times\vec{k}\,\Big)\,\delta(\vec{q}_1-\vec{k})\,, && \text{(A.16)}\\
\vec{\jmath}^{\,(3)}_{FW}(1;\vec{k},\vec{Q}_1)_1 &= -\frac{\vec{Q}_1^{\,2}+\vec{k}^{\,2}}{8m^2}\vec{\jmath}^{\,(1)}_{FW}(1;\vec{k},\vec{Q}_1)_1\\
&\quad -\frac{1}{16m^3}\Big[\Big(\hat{e}_1\,(\vec{k}\cdot\vec{Q}_1)+4\hat{\kappa}_1 m k_0\Big)(\vec{k}+i\vec{\sigma}_1\times\vec{Q}_1)\\
&\quad +\hat{\kappa}_1\,\vec{k}\times\Big[\vec{Q}_1\times(\vec{k}+i\vec{\sigma}_1\times\vec{Q}_1)\Big]\Big]\delta(\vec{q}_1-\vec{k})\,. && \text{(A.17)}
\end{aligned}$$

Note that ATA-(4.1d), corresponding to (A.17), has a wrong power of m and the vector product is missing in the last term. In the second line of (A.17), k_0 stands for the total energy transfer. This means that the corresponding one-nucleon current contains implicitly some interaction effects. The divergence of this part of the current equals the commutator of the full nonrelativistic Hamiltonian with that part of the Darwin-Foldy and spin-orbit charge density of (A.15) which is proportional to $\hat{\kappa}_1$. For the free nucleon one has the relation $k_0=(\vec{k}\cdot\vec{Q}_1)/2m$. It is possible to redefine the one-nucleon current by replacing k_0 by its free-nucleon value or, alternatively, introducing the full momentum transfer also in the preceeding term proportional to $\hat{e}_1$. In this case, the MECs have to be redefined consistenly [3]. In this paper we adopt the particular form (A.17), since then the corresponding MECs have the simplest form.

In order to get the currents in the Breit frame, one simply replaces $\vec{Q}_1$ by $\vec{Q}$, and the δ-function in (A.13) then becomes $\delta(\frac{\vec{k}}{2}-\vec{q})$. Furthermore, for the $(1\leftrightarrow 2)$-exchange one has to make the replacements $\vec{Q}\to-\vec{Q}$ and $\delta(\frac{\vec{k}}{2}-\vec{q})\to\delta(\frac{\vec{k}}{2}+\vec{q})$.

Meson exchange currents

The various contributions from the exchange of a given meson type to the OBE potential are derived in [3] for an arbitrary reference frame. Here we need only the intrinsic ($\vec{P}$-independent) parts of the potential. Their momentum representation is denoted by $\tilde{v}_B(\vec{q},\vec{Q})$ where the isospin dependence has been separated. Thus, up to the order considered one has

$$\tilde{v}_B(\vec{q},\vec{Q}) = \tilde{v}_B^{(1)}(\vec{q}\,) + \tilde{v}_B^{(3)}(\vec{q},\vec{Q}) + \mathcal{O}(m^{-5})\,, \tag{A.18}$$

where $\tilde{v}_B^{(1)}(\vec{q}\,)$ is an even function of $\vec{q}$.

Unlike in [3], we include here explicitly the hadronic form factors into the potentials and exchange currents. This is done by modifying expressions containing a single propagator of a meson of mass m_B by

$$\Delta_B(z) = \frac{1}{m_B^2+z} \rightarrow \tilde{\Delta}_B(z) = \frac{f_B^2(z)}{m_B^2+z}\,, \tag{A.19}$$

$$\Delta_B'(z) = -\frac{1}{(m_B^2+z)^2} \rightarrow \tilde{\Delta}_B'(z) = \frac{d}{d\,z}\,\tilde{\Delta}_B(z)\,, \tag{A.20}$$

where z stands for $\vec{q}^{\,2}$ and f_B is the strong form factor at the meson nucleon vertex.

For the meson-in-flight or mesonic currents the introduction of a hadronic form factor consistent with gauge invariance is achieved by the replacement

$$\Delta_B(z_1)\Delta_B(z_2) \rightarrow f_B(z_1,\,z_2) = -\,\frac{1}{z_1-z_2}\Big(\tilde{\Delta}_B(z_1)-\tilde{\Delta}_B(z_2)\Big)\,, \tag{A.21}$$

where z_i stands for $\vec{q}_i^{\;2}$ $(i=1,\,2)$. This is the minimal form which fulfills the continuity equation [7]. In the case that we assume the hadronic form factor to be of the form

$$f_B(z) = \Big(\frac{\Lambda^2-m_B^2}{\Lambda^2+z}\Big)^{n/2},\ n=1,2,\ldots \tag{A.22}$$

where Λ is a range parameter, it can be shown that then (A.21) corresponds to the minimal substitution in the meson propagator yielding

$$\begin{aligned} f_B(z_1,\,z_2) = {} & (-)^{n-1}\frac{(\Lambda^2-m_B^2)^n}{(n-1)!} \\ & \frac{d^{n-1}}{d(\Lambda^2)^{n-1}}\Big[\frac{1}{\Lambda^2-m_B^2}\Big(\frac{1}{z_1+m_B^2}\frac{1}{z_2+m_B^2}-\frac{1}{z_1+\Lambda^2}\frac{1}{z_2+\Lambda^2}\Big)\Big]\,. \end{aligned} \tag{A.23}$$

All mesonic currents are of nonrelativistic order. In order to get higher order terms, one has to consider the higher order contributions to the meson-nucleon vertices and the effects of retardation of the exchanged mesons, i.e., dependence of the propagator function on the energies transferred. For the general expression (A.21) this means using the Taylor decomposition

$$\begin{aligned} f_B(z_1-\delta_1, z_2-\delta_2) = f_B(z_1,z_2) + \frac{1}{(z_1-z_2)}\Big((\delta_1-\delta_2) f_B(z_1,z_2) \\ +\delta_1\tilde{\Delta}'_B(z_1) - \delta_2\tilde{\Delta}'_B(z_2)\Big) + \mathcal{O}(\delta^2)\,, \end{aligned} \tag{A.24}$$

where $\delta_i = q_{i0}^2$. This relation generalizes ATA-(4.10e). The first term on the r.h.s. of (A.24) contributes to the nonrelativistic mesonic current, the second one also has a propagator structure of the mesonic current and thus will be added to the relativistic "mes" part. It can be shown that the divergence of the corresponding current saturates the commutator of the kinetic energy with the mesonic charge density. The last terms have to be combined with the retardation currents.

The MECs as derived in [3] are associated with particular relativistic Feynman diagrams: nucleon Born, contact, and mesonic contribution. In other techniques, the same currents formally arise from a different set of (time-ordered) diagrams. Therefore we rather group the currents according to their propagator structure, this also allows us to combine several contributions and to present the results in a more compact form. The MECs with a single meson propagator $\tilde{\Delta}_B(\vec{q}_2^{\,2})$ attached are labelled "pro", those corresponding to meson-in-flight currents containing $f_B(\vec{q}_1^{\,2}, \vec{q}_2^{\,2})$ "mes", and finally those containing the derivative of a propagator "ret", since they arise from the retardation or boost contributions. The "pro" and "ret" operators can be separated according to which of the two nucleons interacts with the e.m. field. We always give only the terms corresponding to the e.m. interaction of the first nucleon.

The retardation of the exchanged mesons gives rise to contributions both to the potential and to the MECs. The different prescriptions of how to treat the retardation yield different, but unitarily equivalent results [2]. The unitary transformation is acting in the intrinsic

space only. This unitary equivalence can be parametrized by a parameter ν. The value $\nu = 1/2$ corresponds to the nonretardation potential in the c.m. frame, but it is impossible to find a suitable value for ν which would simplify *all* MECs. Nevertheless, the representation corresponding to $\nu = 1/2$ is most frequently used and we give all intrinsic operators for this choice. For completeness, we list the additional operators for a general ν in Appendix D where we also have corrected several misprints of [3]. For this reason, we will directly refer to this appendix with respect to the "ret" contributions.

For pseudoscalar meson exchange, there is an additional unitary freedom related to various off-energy shell continuations of the energy dependence of the πNN vertex. The corresponding unitary transformation is parametrized by a parameter $\tilde{\mu}$. The simplest form for potential and MEC results from the choice $\tilde{\mu} = 0$, and we have adopted this choice in the main body of this paper. However, the widely used OBEPQ Bonn potentials correspond to the value $\tilde{\mu} = -1$ [18]. Therefore, we present the additional $\tilde{\mu}$-dependent operators in Appendix A.

We have already mentioned in Sect. 3, that the MECs simplify for all exchanges if a part of the relativistic one-nucleon current density is considered to contain interaction effects through its dependence on the total energy transfer k_0 (see (A.17)). Then, the currents, denoted as "external" in [3], are implicitly included and should not be added to the MECs (see section 5 of [3]).

For simplicity, all MECs are multiplied by the Dirac e.m. form factor F_1. For this reason, one has to replace in the explicit expressions of [3] the term $-iF_B^{mes/con}(\vec{\tau}_1 \times \vec{\tau}_2)^3$ by $F_{e,1}^-$. It is shown in the appendix B of [3] how one can modify the MECs in the $1/m$ expansion framework in order to incorporate independent e.m. form factors as suggested in [8]. The resulting additional currents of ATA-(B.17) are proportional to the differences $F_{\gamma BB} - F_1^V$ or $F_{\gamma NNB} - F_1^V$. These differences are of the order of $\vec{k}^{\,2}/\Lambda^2$, where Λ is the corresponding cut-off parameter. The currents ATA-(B.17) are therefore of leading relativistic order. Moreover, they are frame-independent and therefore can be added to the intrinsic currents considered in this paper without any further modification.

With respect to our notation, we would like to remark that the currents in the momentum space representation depend in general on $\vec{k}$, $\vec{K}$, $\vec{q}$, and $\vec{Q}$, but for the sake of briefness only their $\vec{k}$-dependence is retained in our notation of the operators while in the explicit expressions on the r.h.s. of the equations we use $\vec{k}$, $\vec{q}_{1/2} = \frac{1}{2}\vec{k} \pm \vec{q}$, and $\vec{Q}_{1/2} = \frac{1}{2}\vec{K} \pm \vec{Q}$ for convenience.

Scalar meson exchange

With respect to the "+" part of the MECs for scalar exchange, there is one single nonretardation current from ATA-(4.4b)

$$\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{pro};\vec{k}\,)^{+}_{S} = -F^{+}_{e,1}\frac{1}{4m^2}\tilde{v}^{(1)}_{S}(\vec{q}_2\,)(\vec{Q}_1 + i\vec{\sigma}_1 \times \vec{k}) + (1 \leftrightarrow 2)\,, \tag{A.25}$$

where $\tilde{v}^{(1)}_{S}(\vec{q}_2\,)$ is given in (75). Furthermore, one gets the retardation charge and current densities from (D.8)-(D.10) for $\tilde{\nu} = 0$

$$\rho^{(2)}_{FW}(2;\mathrm{ret};\vec{k}\,)^{+}_{S} = F^{+}_{e,1}\,\frac{g_S^2}{(2\pi)^3\,8m}\,(\vec{k}\cdot\vec{q}_2)\,\tilde{\Delta}'_S(\vec{q}_2^{\,2}) + (1 \leftrightarrow 2)\,, \tag{A.26}$$

$$\begin{aligned}\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{ret};\vec{k}\,)^{+}_{S} = {}& \frac{g_S^2}{(2\pi)^3\,16m^2}\Big\{F^{+}_{e,1}\left(\vec{Q}_1\,(\vec{k}\cdot\vec{q}_2) + \vec{q}_2\,(\vec{Q}_1 + 3\vec{Q}_2)\cdot\vec{q}_2\right)\\ & +iG^{+}_{M,1}\,\vec{\sigma}_1 \times \vec{k}\,(\vec{k}\cdot\vec{q}_2)\Big\}\,\tilde{\Delta}'_S(\vec{q}_2^{\,2}) + (1 \leftrightarrow 2)\,.\end{aligned} \tag{A.27}$$

Similarly, one obtains for the the "−" part from ATA-(4.3b) one nonrelativistic current

$$\vec{\jmath}^{\,(1)}_{FW}(2;\mathrm{mes};\vec{k}\,)^{-}_{S} = F^{-}_{e,1}\frac{g_S^2}{(2\pi)^3}\,(\vec{q}_1 - \vec{q}_2)\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,. \tag{A.28}$$

The relativistic "pro" and "mes" contributions follow from ATA-(4.4b, 4.7d, 4.10a)

$$\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{pro};\vec{k}\,)^{-}_{S} = F^{-}_{e,1}\frac{1}{4m^2}\tilde{v}^{(1)}_{S}(\vec{q}_2\,)(\vec{k} + i\vec{\sigma}_1 \times \vec{Q}_1) + (1 \leftrightarrow 2)\,, \tag{A.29}$$

$$\rho^{(2)}_{FW}(2;\mathrm{mes};\vec{k}\,)^{-}_{S} = F^{-}_{e,1}\frac{g_S^2}{(2\pi)^3}\,(q_{10} - q_{20})\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,, \tag{A.30}$$

$$\begin{aligned}\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{mes};\vec{k}\,)^{-}_{S} = {}& -F^{-}_{e,1}\,\frac{g_S^2}{(2\pi)^3\,8m^2}f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,(\vec{q}_1 - \vec{q}_2\,)\Big(\vec{Q}_1^{\,2} + \vec{Q}_2^{\,2}\\ & +i\vec{\sigma}_1\cdot(\vec{q}_1 \times \vec{Q}_1) + i\vec{\sigma}_2\cdot(\vec{q}_2 \times \vec{Q}_2) - 8m^2\frac{q_{10}^2 - q_{20}^2}{\vec{q}_1^{\,2} - \vec{q}_2^{\,2}}\Big)\,,\end{aligned} \tag{A.31}$$

while again the "ret" currents come from (D.8)-(D.10) for $\tilde{\nu} = 0$

$$\rho^{(2)}_{FW}(2;\mathrm{ret};\vec{k}\,)^-_S = -F^-_{e,1}\frac{g_S^2}{(2\pi)^3\,8m}\,(\vec{Q}_1 + 3\vec{Q}_2)\cdot\vec{q}_2\;\tilde{\Delta}'_S(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,. \tag{A.32}$$

$$\begin{aligned}\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{ret};\vec{k}\,)^-_S &= -\frac{g_S^2}{(2\pi)^3\,16m^2}\Big\{F^-_{e,1}\Big(\vec{q}_2\,(\vec{k}\cdot\vec{q}_2) + \vec{Q}_1\,(\vec{Q}_1 + 3\vec{Q}_2)\cdot\vec{q}_2\Big)\\ &\quad + iG^-_{M,1}\,\vec{\sigma}_1\times\vec{k}\;(\vec{Q}_1 + 3\vec{Q}_2)\cdot\vec{q}_2 + F^-_{e,1}\,16m^2\frac{\vec{q}_1 - \vec{q}_2}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})}\,q_{20}^2\Big\}\tilde{\Delta}'_S(\vec{q}_2^{\,2})\\ &\quad + (1\leftrightarrow 2)\,.\end{aligned} \tag{A.33}$$

Vector meson exchange

Large parts of the MEC for vector meson exchange can be obtained from the scalar case by the replacements $m_S \to m_V$, $g_S^2 \to -g_V^2$. Those parts will not be listed again, rather we shall refer to the corresponding expressions for scalar exchange keeping in mind the aforementioned replacements. Only the additional currents will now be listed explicitly.

We start again with the "+" part for which we have only one additional current from ATA-(4.4c)

$$\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{pro};\vec{k}\,)^+_V = -F^+_{e,1}\frac{1}{4m^2}\tilde{v}^{(1)}_V(\vec{q}_2\,)\Big(\vec{Q}_2 + i(1+\kappa_V)\,(\vec{\sigma}_1 + \vec{\sigma}_2)\times\vec{q}_2\Big) + (1\leftrightarrow 2)\,, \tag{A.34}$$

where κ_V is the usual ratio of the normal (vector) to the anomalous (tensor) VNN coupling constants. The retardation currents follow from (A.26) and (A.27).

For the "$-$" part, we have first the retardation terms from (A.32) and (A.33). For the additional currents we collect from the expressions in ATA-(4.4c, 4.9a)

$$\begin{aligned}\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{pro};\vec{k}\,)^-_V &= F^-_{e,1}\frac{1}{4m^2}\tilde{v}^{(1)}_V(\vec{q}_2\,)\Big((1+\kappa_V)\vec{q}_2 - \kappa_V\vec{q}_1 - (1+\kappa_V)^2\,\vec{\sigma}_1\times(\vec{\sigma}_2\times\vec{q}_2)\\ &\quad - i\kappa_V\vec{\sigma}_1\times\vec{Q}_1 + i(1+\kappa_V)\vec{\sigma}_1\times\vec{Q}_2\Big) + (1\leftrightarrow 2)\,.\end{aligned} \tag{A.35}$$

For the "mes" currents, one has two contributions from ATA-(4.7c, 4.8). First from (A.30) the charge density $\rho^{(2)}_{FW}(2;\mathrm{mes};\vec{k}\,)^-_V$, and from ATA-(4.10b) plus the last term of (A.31) the current density

$$\vec{\jmath}^{(3)}_{FW}(2;\text{mes};\vec{k}\,)^{-}_{V} = F^{-}_{e,1}\,\frac{g_V^2}{(2\pi)^3\,4m^2}\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,(\vec{q}_1-\vec{q}_2\,)$$
$$\Big[(\frac{1}{2}+\kappa_V)(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})+(\vec{Q}_1\cdot\vec{Q}_2)-(1+\kappa_V)^2(\vec{\sigma}_1\times\vec{q}_1)\cdot(\vec{\sigma}_2\times\vec{q}_2)$$
$$+i(\frac{1}{2}+\kappa_V)\Big(\vec{\sigma}_1\cdot(\vec{Q}_1\times\vec{q}_1)+\vec{\sigma}_2\cdot(\vec{Q}_2\times\vec{q}_2)\Big)$$
$$-i(1+\kappa_V)\Big(\vec{\sigma}_1\cdot(\vec{Q}_2\times\vec{q}_1)+\vec{\sigma}_2\cdot(\vec{Q}_1\times\vec{q}_2)\Big)-4m^2\frac{q_{10}^2-q_{20}^2}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}\Big]\,. \qquad \text{(A.36)}$$

Second, from the transverse part of the γVV vertex as given in ATA-(4.7e, 4.10c)

$$\rho^{(2)}_{FW}(2;\text{mes-tr};\vec{k}\,)^{-}_{V} = F^{-}_{e,1}\,\frac{g_V^2}{(2\pi)^3\,2m}\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})$$
$$\Big(\vec{k}\cdot(\vec{Q}_1-\vec{Q}_2)+i(1+\kappa_V)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}_1\times\vec{q}_2)\Big)\,, \qquad \text{(A.37)}$$
$$\vec{\jmath}^{(3)}_{FW}(2;\text{mes-tr};\vec{k}\,)^{-}_{V} = F^{-}_{e,1}\,\frac{g_V^2}{(2\pi)^3\,4m^2}\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})$$
$$\Big\{2mk_0\Big[\vec{Q}_1-\vec{Q}_2+i(1+\kappa_V)(\vec{\sigma}_1\times\vec{q}_1-\vec{\sigma}_2\times\vec{q}_2)\Big]$$
$$+\vec{k}\times\Big[(1+\kappa_V)^2\,(\vec{\sigma}_1\times\vec{q}_1)\times(\vec{\sigma}_2\times\vec{q}_2)-\vec{Q}_1\times\vec{Q}_2$$
$$+i(1+\kappa_V)\Big((\vec{\sigma}_2\times\vec{q}_2)\times\vec{Q}_1-(\vec{\sigma}_1\times\vec{q}_1)\times\vec{Q}_2\Big)\Big]\Big\}\,. \qquad \text{(A.38)}$$

The usual minimal form of this part of the vertex is adopted here, in accordance with ATA. The discussion and implications of a nonminimal form can be found in the appendix B of ATA and in [8].

Pseudoscalar meson exchange

In the case of pseudoscalar meson exchange, a unitary freedom shows up in the potential and in the interaction currents which is not present for scalar and vector exchange up to the order considered here. Let us recall that in order to derive the nuclear potential and e.m. current operators from a meson-nucleon Lagrangian, one has to fix the energy transfer at the meson-nucleon vertices. This is done in different ways within various techniques [2]. In particular, in [3] the vertex energy transfer was replaced by the difference of the on-mass-shell nucleon energies. For the off-energy-shell parameter β introduced in [18] this means $\beta = 0$. Most of the other techniques, e.g. those of [13,16], correspond to a symmetric

off-energy-shell continuation with $\beta = 1/2$. Whatever choice is adopted, the final results are unitarily equivalent [2], the unitary parameter $\tilde{\mu}$ being given by

$$\tilde{\mu} = 4\beta\Big(\frac{1-c}{2}(\mu-1)+1\Big) - 1\,, \tag{A.39}$$

where $\mu = 0\,(1)$ for $PS(PV)$ πNNcoupling, and c is the so-called Barnhill parameter (for a more detailed discussion see [2,13,16,18]).

This unitary freedom does not affect the nonrelativistic parts of the potential and the MEC but the leading order relativistic contributions. It means that $\tilde{v}^{(3)}$ and $j_\lambda^{(3)}(2;k)$ for an arbitrary choice of the parameter $\tilde{\mu}$ are obtained from the operators for $\tilde{\mu} = 0$ by the approximate unitary transformation $(1 - i\tilde{\mu}U_{PS})$ where

$$\langle \vec{p}_1^{\,\prime}\vec{p}_2^{\,\prime}\,|\,U_{PS}\,|\vec{p}_1\vec{p}_2\,\rangle = \tilde{U}_{PS}(\vec{q},\vec{K},\vec{Q})(\vec{\tau}_1\cdot\vec{\tau}_2)\,\delta(\vec{q}_1+\vec{q}_2\,)\,, \tag{A.40}$$

$$i\tilde{U}_{PS}(\vec{q},\vec{K},\vec{Q}) = \frac{g_{PS}^2}{(2\pi)^3\,32m^3}\Big[\frac{1}{2}\Sigma^{(-)}(\vec{q},\,\vec{K}) - \Sigma^{(+)}(\vec{q},\,\vec{Q})\Big]\,\tilde{\Delta}_{PS}(\vec{q}^{\,2})\,, \tag{A.41}$$

where we have introduced as abbreviation for two vectors $\vec{a}$ and $\vec{b}$

$$\Sigma^{(\pm)}(\vec{a},\,\vec{b}) = (\vec{\sigma}_1\cdot\vec{a})(\vec{\sigma}_2\cdot\vec{b}) \pm (\vec{\sigma}_1\cdot\vec{b})(\vec{\sigma}_2\cdot\vec{a})\,. \tag{A.42}$$

Note that here $\vec{P}^{\,\prime} = \vec{P}$ and $\vec{K} = 2\vec{P} = 2(\vec{p}_1+\vec{p}_2)$. In detail one finds

$$\tilde{v}^{(3)}_{PS(\tilde{\mu})}(\vec{q},\vec{Q}) = \tilde{v}^{(3)}_{PS}(\vec{q},\vec{Q}) + \tilde{v}^{(3)}_{\tilde{\mu}}(\vec{q},\vec{Q})\,, \tag{A.43}$$

where

$$\begin{aligned}\tilde{v}^{(3)}_{\tilde{\mu}}(\vec{q},\vec{Q}) &= i\tilde{\mu}\langle\vec{p}_1^{\,\prime}\vec{p}_2^{\,\prime}\,|[t^{(1)},\,U_{PS}]|\vec{p}_1\vec{p}_2\,\rangle \\ &= -\tilde{\mu}\frac{g_{PS}^2}{(2\pi)^3\,32m^4}(\vec{q}\cdot\vec{Q})\Sigma^{(+)}(\vec{q},\,\vec{Q})\;\tilde{\Delta}_{PS}(\vec{q}^{\,2})\,,\end{aligned} \tag{A.44}$$

and

$$j_{\lambda,FW}(2;\tilde{\mu};\vec{k}\,)_{PS} = j_{\lambda,FW}(2;\vec{k}\,)_{PS} + j_{\lambda,FW\,\tilde{\mu}}(2;\vec{k}\,)_{PS}\,, \tag{A.45}$$

with

$$j_{\lambda,FW\,\tilde{\mu}}(2;\vec{k}\,)_{PS} = i\tilde{\mu}\Big[j_{\lambda,FW}(1;\vec{k}\,),\,U_{PS}\Big]\,. \tag{A.46}$$

Evaluating (A.46) with (A.41), one finds for the additional $\tilde\mu$-dependent FW currents

$$\begin{aligned}
\rho^{(2)}_{FW\,\tilde\mu}(2;\vec k\,)_{PS} = & -\tilde\mu\frac{g^2_{PS}}{(2\pi)^3\,32m^3}\Big\{\,F^+_{e,1}\,(\vec\sigma_1\cdot\vec k)(\vec\sigma_2\cdot\vec q_2) \\
& +F^-_{e,1}\Big[(\vec\sigma_1\cdot\vec q_2)(\vec\sigma_2\cdot\vec Q_2)-(\vec\sigma_1\cdot\vec Q_1)(\vec\sigma_2\cdot\vec q_2)\Big]\Big\}\,\tilde\Delta_{PS}(\vec q_2^{\,2})+(1\leftrightarrow 2)\,, \quad (A.47) \\
\vec\jmath^{\,(3)}_{FW\,\tilde\mu}(2;\vec k\,)_{PS} = & -\tilde\mu\frac{g^2_{PS}}{(2\pi)^3\,64m^4} \\
& \Big\{\,F^+_{e,1}\Big[\vec Q_1(\vec\sigma_1\cdot\vec k)(\vec\sigma_2\cdot\vec q_2)-\vec q_2\Big((\vec\sigma_1\cdot\vec q_2)(\vec\sigma_2\cdot\vec Q_2)-(\vec\sigma_1\cdot\vec Q_1)(\vec\sigma_2\cdot\vec q_2)\Big)\Big] \\
& +G^+_{M,1}\,\vec k\times\Big[\vec q_2\times\vec\sigma_1(\vec\sigma_2\cdot\vec Q_2)-\vec Q_1\times\vec\sigma_1(\vec\sigma_2\cdot\vec q_2)\Big] \\
& +F^-_{e,1}\Big[\vec Q_1\Big\{(\vec\sigma_1\cdot\vec q_2)(\vec\sigma_2\cdot\vec Q_2)-(\vec\sigma_1\cdot\vec Q_1)(\vec\sigma_2\cdot\vec q_2)\Big\}-\vec q_2\,(\vec\sigma_1\cdot\vec k)(\vec\sigma_2\cdot\vec q_2)\Big] \\
& +G^-_{M,1}\,\vec k\times\Big[\vec k\times\vec\sigma_1(\vec\sigma_2\cdot\vec q_2)+i\vec Q_1(\vec\sigma_2\cdot\vec q_2)-i\vec q_2(\vec\sigma_2\cdot\vec Q_2)\Big]\Big\}\,\tilde\Delta_{PS}(\vec q_2^{\,2}) \\
& +(1\leftrightarrow 2)\,. \quad (A.48)
\end{aligned}$$

After these remarks concerning the unitary freedom, we will now proceed to give the explicit expressions for the FW currents. We will start with those given in [3] which correspond to the unitary representation $\tilde\mu=-1$ [2,18]. However, this unitary freedom affects the "pro"-currents only for which we will characterize the expressions from [3] by an additional subscript "ATA". The FW currents for the unitary representation $\tilde\mu=0$ are then obtained by adding (A.47) or (A.48), respectively, for $\tilde\mu=1$ to the corresponding expressions, i.e.,

$$\rho^{(2)}_{FW}(2;\mathrm{pro};\vec k\,)^{\pm}_{PS}=\rho^{(2)}_{FW}(2;\mathrm{pro};\vec k\,)^{\pm}_{PS,\,ATA}+\rho^{(2)}_{FW\,\tilde\mu=1}(2;\vec k\,)^{\pm}_{PS}\,, \qquad (A.49)$$

$$\vec\jmath^{\,(3)}_{FW}(2;\mathrm{pro};\vec k\,)^{\pm}_{PS}=\vec\jmath^{\,(3)}_{FW}(2;\mathrm{pro};\vec k\,)^{\pm}_{PS,\,ATA}+\vec\jmath^{\,(3)}_{FW\,\tilde\mu=1}(2;\vec k\,)^{\pm}_{PS}\,. \qquad (A.50)$$

All other FW currents are $\tilde\mu$-independent.

Considering first the "+" terms, their "pro" part follows from ATA-(4.4d) and (4.19a,b). Explicitly, one finds

$$\rho^{(2)}_{FW}(2;\mathrm{pro};\vec k\,)^{+}_{PS,\,ATA}=F^+_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,8m^3}\,(\vec\sigma_1\cdot\vec k)(\vec\sigma_2\cdot\vec q_2)\;\;\tilde\Delta_{PS}(\vec q_2^{\,2})+(1\leftrightarrow 2)\,, \qquad (A.51)$$

$$\begin{aligned}
\vec\jmath^{\,(3)}_{FW}(2;\mathrm{pro};\vec k\,)^{+}_{PS,\,ATA}= & \frac{g^2_{PS}}{(2\pi)^3\,16m^4}\Big\{F^+_{e,1}\Big[\vec Q_1(\vec\sigma_1\cdot\vec q_2)+\vec\sigma_1(\vec k\cdot\vec Q_1)+\vec\sigma_1(\vec q_2\cdot\vec Q_2)-i(\vec k\times\vec q_2)\Big] \\
& +F^+_{\kappa,1}\,\vec k\times(\vec\sigma_1\times\vec Q_1)\Big\}(\vec\sigma_2\cdot\vec q_2)\;\;\tilde\Delta_{PS}(\vec q_2^{\,2})+(1\leftrightarrow 2)\,. \qquad (A.52)
\end{aligned}$$

The transformations (A.49) and (A.50) give then the FW-"pro"-current for $\tilde\mu=0$

$$\rho^{(2)}_{FW}(2;\text{pro};\vec{k}\,)^{+}_{PS} = F^{+}_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^3}\,3\,(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\vec{q}_2)\;\;\tilde{\Delta}_{PS}(\vec{q}^{\,2}_2) + (1\leftrightarrow 2)\,, \qquad \text{(A.53)}$$

$$\begin{aligned}\vec{\jmath}^{\,(3)}_{FW}(2;\text{pro};\vec{k}\,)^{+}_{PS} = \frac{g^2_{PS}}{(2\pi)^3\,16m^4}\Big\{&F^{+}_{e,1}\,\Big(\frac{1}{4}\vec{q}_2\,\big[(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{Q}_2) - (\vec{\sigma}_1\cdot\vec{Q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)\big]\\ &+\Big[\vec{Q}_1(\vec{\sigma}_1\cdot(\vec{q}_2+\tfrac{3}{4}\vec{k}\,)) + \vec{\sigma}_1(\vec{q}_2\cdot\vec{Q}_2) - i(\vec{k}\times\vec{q}_2)\Big)\Big](\vec{\sigma}_2\cdot\vec{q}_2)\,\Big)\\ &+\frac{1}{4}G^{+}_{M,1}\,\vec{k}\times\Big[3\,\vec{\sigma}_1\times\vec{Q}_1\,(\vec{\sigma}_2\cdot\vec{q}_2) - \vec{\sigma}_1\times\vec{q}_2\,(\vec{\sigma}_2\cdot\vec{Q}_2)\Big]\Big\}\;\tilde{\Delta}_{PS}(\vec{q}^{\,2}_2)\\ &+(1\leftrightarrow 2)\,. \qquad \text{(A.54)}\end{aligned}$$

The "ret" part we take from (D.8) through (D.13) of Appendix D.

$$\rho^{(2)}_{FW}(2;\text{ret};\vec{k}\,)^{+}_{PS} = F^{+}_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^3}\,(\vec{k}\cdot\vec{q}_2)(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2)\,\tilde{\Delta}'_{PS}(\vec{q}^{\,2}_2) + (1\leftrightarrow 2)\,, \qquad \text{(A.55)}$$

$$\begin{aligned}\vec{\jmath}^{\,(3)}_{FW}(2;\text{ret};\vec{k}\,)^{+}_{PS} = \frac{g^2_{PS}}{(2\pi)^3\,64m^4}\Big\{&F^{+}_{e,1}\,(\vec{\sigma}_1\cdot\vec{q}_2)\Big[\vec{Q}_1\,(\vec{k}\cdot\vec{q}_2) + \vec{q}_2\,(\vec{Q}_1+3\vec{Q}_2)\cdot\vec{q}_2\,\Big]\\ &+G^{+}_{M,1}\,\vec{k}\times\;\Big[\vec{\sigma}_1\times\vec{q}_2\,(\vec{Q}_1+3\vec{Q}_2)\cdot\vec{q}_2 - i\vec{q}_2\,(\vec{k}\cdot\vec{q}_2)\Big]\Big\}(\vec{\sigma}_2\cdot\vec{q}_2)\,\tilde{\Delta}'_{PS}(\vec{q}^{\,2}_2)\\ &+(1\leftrightarrow 2)\,. \qquad \text{(A.56)}\end{aligned}$$

With respect to the "$-$" part, there are first of all two well-known $\tilde{\mu}$-independent non-relativistic currents from ATA-(4.3d,e), namely

$$\vec{\jmath}^{\,(1)}_{FW}(2;\text{pro};\vec{k}\,)^{-}_{PS} = F^{-}_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,4m^2}\,\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{q}_2)\;\tilde{\Delta}_{PS}(\vec{q}^{\,2}_2) + (1\leftrightarrow 2)\,, \qquad \text{(A.57)}$$

$$\vec{\jmath}^{\,(1)}_{FW}(2;\text{mes};\vec{k}\,)^{-}_{PS} = -F^{-}_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,4m^2}\,(\vec{q}_1-\vec{q}_2)\,(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)f(\vec{q}^{\,2}_1,\vec{q}^{\,2}_2)\,. \qquad \text{(A.58)}$$

For the relativistic MECs the "pro" parts are taken from ATA-(4.4d, 4.7b, 4.9b, 4.19). Notice that that the first "$-$" term in ATA-(4.4d) should read $\vec{\sigma}_1\vec{q}^{\,2}_2$ and the sign in front of the G^{-}-terms in ATA-(4.19b) should be changed. Explicitly, one finds that $\rho^{(2)}_{FW}(2;\text{pro};\vec{k}\,)^{-}_{PS}$ vanishes since ATA-(4.7b) and ATA-(4.19a) cancel exactly. For the current one has

$$\begin{aligned}\vec{\jmath}^{\,(3)}_{FW}(2;\text{pro};\vec{k}\,)^{-}_{PS,\,ATA} = -\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\Big\{&F^{-}_{e,1}\Big[\Big(\vec{\sigma}_1(3\vec{q}^{\,2}_2+\vec{Q}^{\,2}_1+\vec{Q}^{\,2}_2) + 2\vec{k}(\vec{\sigma}_1\cdot\vec{q}_2) + \vec{q}_1(\vec{\sigma}_1\cdot\vec{q}_1)\\ &+\vec{Q}_1(\vec{\sigma}_1\cdot\vec{Q}_1) + i(\vec{k}+\vec{q}_2)\times\vec{Q}_1\Big)(\vec{\sigma}_2\cdot\vec{q}_2) + \vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{Q}_2)(\vec{q}_2\cdot\vec{Q}_2)\Big]\\ &-2G^{-}_{M,1}\,\vec{k}\times\Big(i\vec{Q}_1+\vec{k}\times\vec{\sigma}_1\Big)(\vec{\sigma}_2\cdot\vec{q}_2)\Big\}\;\tilde{\Delta}_{PS}(\vec{q}^{\,2}_2) + (1\leftrightarrow 2)\,. \qquad \text{(A.59)}\end{aligned}$$

Again the transformations (A.49) and (A.50) give then the FW-"pro"-current for $\tilde{\mu}=0$

$$\rho^{(2)}_{FW}(2;\mathrm{pro};\vec{k}\,)^{-}_{PS} = -F^{-}_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^3}\,\Big[(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{Q}_2) - (\vec{\sigma}_1\cdot\vec{Q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)\Big]\;\tilde{\Delta}_{PS}(\vec{q}^{\,2}_2)$$
$$+(1\leftrightarrow 2)\,, \tag{A.60}$$

$$\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{pro};\vec{k}\,)^{-}_{PS} = -\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\Big\{F^{-}_{e,1}\Big[\Big(\vec{\sigma}_1(3\vec{q}^{\,2}_2 + \vec{Q}^{\,2}_1 + \vec{Q}^{\,2}_2) + 2\vec{k}(\vec{\sigma}_1\cdot\vec{q}_2) + \vec{q}_1(\vec{\sigma}_1\cdot\vec{q}_1)$$
$$-\frac{1}{2}\vec{q}_2(\vec{\sigma}_1\cdot\vec{k}) + i(\vec{k}+\vec{q}_2)\times\vec{Q}_1\Big)(\vec{\sigma}_2\cdot\vec{q}_2) + \vec{\sigma}_1(\vec{q}_2\cdot\vec{Q}_2)(\vec{\sigma}_2\cdot\vec{Q}_2)\Big]$$
$$+\frac{1}{2}\vec{Q}_1\Big((\vec{\sigma}_1\cdot\vec{Q}_1)(\vec{\sigma}_2\cdot\vec{q}_2) + (\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{Q}_2)\Big)$$
$$-\frac{1}{2}G^{-}_{M,1}\,\vec{k}\times\Big[3\Big(i\vec{Q}_1 + \vec{k}\times\vec{\sigma}_1\Big)(\vec{\sigma}_2\cdot\vec{q}_2) + i\vec{q}_2(\vec{\sigma}_2\cdot\vec{Q}_2)\Big\}\;\tilde{\Delta}_{PS}(\vec{q}^{\,2}_2)$$
$$+(1\leftrightarrow 2)\,. \tag{A.61}$$

The "mes" currents follow from ATA-(4.7f, 4.10d,e)

$$\rho^{(2)}_{FW}(2;\mathrm{mes};\vec{k}\,)^{-}_{PS} = -F^{-}_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,4m^2}\,(q_{10}-q_{20})\,(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)f(\vec{q}^{\,2}_1,\vec{q}^{\,2}_2)\,, \tag{A.62}$$

$$\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{mes};\vec{k}\,)^{-}_{PS} = F^{-}_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\,f(\vec{q}^{\,2}_1,\vec{q}^{\,2}_2)\,(\vec{q}_1-\vec{q}_2)\Big\{(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{Q}_2)(\vec{q}_2\cdot\vec{Q}_2)$$
$$+(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)\Big[\vec{q}^{\,2}_1 + \vec{Q}^{\,2}_1 - 4m^2\frac{q^2_{10}-q^2_{20}}{\vec{q}^{\,2}_1-\vec{q}^{\,2}_2}\Big] + (1\leftrightarrow 2)\Big\}\,, \tag{A.63}$$

and finally the corrected retardation operators from (D.8)-(D.13) for $\tilde{\nu}=0$

$$\rho^{(2)}_{FW}(2;\mathrm{ret};\vec{k}\,)^{-}_{PS} = -F^{-}_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,32m^3}\,(\vec{Q}_1+3\vec{Q}_2)\cdot\vec{q}_2\,(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2)\;\tilde{\Delta}'_{PS}(\vec{q}^{\,2}_2)$$
$$+(1\leftrightarrow 2)\,, \tag{A.64}$$

$$\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{ret};\vec{k}\,)^{-}_{PS} = -\frac{g^2_{PS}}{(2\pi)^3\,64m^4}\Big\{F^{-}_{e,1}\,\Big[16m^2\,q^2_{20}\,\Big(\vec{\sigma}_1 - \frac{\vec{q}_1-\vec{q}_2}{\vec{q}^{\,2}_1-\vec{q}^{\,2}_2}\,(\vec{\sigma}_1\cdot\vec{q}_1)\,\Big)$$
$$+(\vec{\sigma}_1\cdot\vec{q}_2)\Big(\vec{q}_2\,(\vec{k}\cdot\vec{q}_2) + \vec{Q}_1\,(\vec{Q}_1+3\vec{Q}_2)\cdot\vec{q}_2\,\Big)\Big]$$
$$-G^{-}_{M,1}\,i\vec{k}\times\Big(\vec{q}_2\,(\vec{Q}_1+3\vec{Q}_2)\cdot\vec{q}_2 + i\vec{\sigma}_1\times\vec{q}_2\,(\vec{k}\cdot\vec{q}_2)\,\Big)\Big\}(\vec{\sigma}_2\cdot\vec{q}_2)\tilde{\Delta}'_{PS}(\vec{q}^{\,2}_2)$$
$$+(1\leftrightarrow 2)\,. \tag{A.65}$$

APPENDIX B: BOOST AND SEPARATION CONTRIBUTIONS FOR SCALAR AND PSEUDOSCALAR MESON EXCHANGE CURRENTS

In this appendix, we collect the separation and boost current contributions to the interaction currents for scalar and pseudoscalar exchange only, since for vector exchange they

are formally equal to the scalar ones. The separation and kinematic boost contributions arise from the nonrelativistic meson exchange current whereas a potential dependent boost appears from the one-body current for pseudoscalar exchange only.

Scalar meson exchange

Taking the only nonrelativistic current contribution from (A.28), we find by separating into the "pro", "mes" and "ret" parts

$$\vec{\jmath}^{(3)}_{\chi_\sigma}(2;\text{mes};\vec{k},\vec{q},\vec{Q})^-_S = iF^-_{e,1}\,\frac{g_S^2}{(2\pi)^3\,8m^2}\,\vec{q}\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{Q}\times\vec{k})\,, \tag{B.1}$$

$$\vec{\jmath}^{(3)}_{\chi_r}(2;\text{pro};\vec{k},\vec{q},\vec{Q})^-_S = -F^-_{e,1}\,\frac{g_S^2}{(2\pi)^3\,32m^2}\,\vec{k}\;\tilde{\Delta}_S(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{B.2}$$

$$\vec{\jmath}^{(3)}_{\chi_r}(2;\text{ret};\vec{k},\vec{q},\vec{Q})^-_S = -F^-_{e,1}\,\frac{g_S^2}{(2\pi)^3\,16m^2}\,\vec{q}\,(\vec{k}\cdot\vec{q}_2)\,\tilde{\Delta}'_S(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{B.3}$$

$$\vec{\jmath}^{(3)}_{sep}(2;\text{ret};\vec{k},\vec{q},\vec{Q})^-_S = F^-_{e,1}\,\frac{g_S^2}{(2\pi)^3\,32m^2}\,\vec{q}\,\vec{k}^{\,2}\,\frac{(\vec{k}\cdot\vec{q}_2)}{(\vec{k}\cdot\vec{q}\,)}\tilde{\Delta}'_S(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,. \tag{B.4}$$

Pseudoscalar meson exchange

We will list first the contributions from the kinematic boost and start with the χ_σ-currents with separation according to the propagator structure

$$\begin{aligned}\vec{\jmath}^{(3)}_{\chi_\sigma}(2;\text{pro};\vec{k},\vec{q},\vec{Q})^-_{PS} = {}& F^-_{e,1}\,\frac{g_{PS}^2}{(2\pi)^3\,64m^4}\\ &\Big\{\Big[\vec{k}\,(\vec{\sigma}_1\cdot\vec{q}\,)-i(\vec{k}\times\vec{Q})\Big](\vec{\sigma}_2\cdot\vec{q}_2)-\vec{q}\,(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{k})\\ &+\vec{\sigma}_1\Big[(\vec{\sigma}_2\times\vec{q}_2)\cdot(\vec{k}\times\vec{q}\,)+i(\vec{k}\times\vec{q}\,)\cdot\vec{Q}\Big]\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,,\end{aligned} \tag{B.5}$$

$$\begin{aligned}\vec{\jmath}^{(3)}_{\chi_\sigma}(2;\text{mes};\vec{k},\vec{q},\vec{Q})^-_{PS} = {}& F^-_{e,1}\,\frac{g_{PS}^2}{(2\pi)^3\,32m^4}\vec{q}\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\Big\{\vec{q}^{\,2}(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\vec{k})+\vec{k}^{\,2}(\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}\,)\\ &-(\vec{k}\times\vec{q}\,)\cdot\Big[i\vec{Q}\,(\vec{\sigma}_1\cdot\vec{q}_1+\vec{\sigma}_2\cdot\vec{q}_2)-(\vec{\sigma}_1\times\vec{\sigma}_2)(\vec{q}_1\cdot\vec{q}_2)\Big]\,.\end{aligned} \tag{B.6}$$

Analogously, we get from (43) and (29) for the χ_r- and separation currents

$$\vec{\jmath}^{(3)}_{\chi_r}(2;\text{pro};\vec{k},\vec{q},\vec{Q})^-_{PS} = F^-_{e,1}\,\frac{g_{PS}^2}{(2\pi)^3\,128m^4}\Big\{\vec{\sigma}_1\Big(\vec{k}^{\,2}(\vec{\sigma}_2\cdot\vec{q}_2)-(\vec{k}\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{k})\Big)$$

$$-\vec{q}\Big((\vec{\sigma}_1\cdot\vec{k}\,)(\vec{\sigma}_2\cdot\vec{q}_2)-(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{k}\,)\Big)$$

$$-\vec{k}\,(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)\Big\}\ \tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{B.7}$$

$$\vec{\jmath}^{\,(3)}_{\chi_r}(2;\mathrm{ret};\vec{k},\vec{q},\vec{Q})^-_{PS}=-F^-_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,64m^4}\Big[\vec{\sigma}_1-\vec{q}\,\frac{(\vec{\sigma}_1\cdot\vec{q}_1)}{(\vec{k}\cdot\vec{q}\,)}\Big]$$

$$(\vec{\sigma}_2\cdot\vec{q}_2)(\vec{k}\cdot\vec{q}\,)(\vec{k}\cdot\vec{q}_2)\tilde{\Delta}'_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{B.8}$$

$$\vec{\jmath}^{\,(3)}_{sep}(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})^-_{PS}=F^-_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,128m^4}\vec{\sigma}_1\,\vec{k}^{\,2}(\vec{\sigma}_2\cdot(\vec{k}-\vec{q}\,))\ \tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{B.9}$$

$$\vec{\jmath}^{\,(3)}_{sep}(2;\mathrm{mes};\vec{k},\vec{q},\vec{Q})^-_{PS}=-F^-_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,128m^4}\,\vec{q}\,\vec{k}^{\,2}f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})$$

$$\Big[(\vec{\sigma}_1\cdot\vec{k}\,)(\vec{\sigma}_2\cdot\vec{q}_2)+(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{k}\,)\Big]\,, \tag{B.10}$$

$$\vec{\jmath}^{\,(3)}_{sep}(2;\mathrm{ret};\vec{k},\vec{q},\vec{Q})^-_{PS}=F^-_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,128m^4}\Big[\vec{\sigma}_1-\vec{q}\,\frac{(\vec{\sigma}_1\cdot\vec{q}_1)}{(\vec{k}\cdot\vec{q}\,)}\Big]$$

$$(\vec{\sigma}_2\cdot\vec{q}_2)\,\vec{k}^{\,2}\,(\vec{k}\cdot\vec{q}_2)\tilde{\Delta}'_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{B.11}$$

where we have again separated the contributions with respect to their propagator structure.

Finally, we list the interaction dependent boost contribution of the one-body current

$$j_{\lambda,\chi_V}(2;\vec{k}\,)_{PS}=-i\Big[j_{\lambda,FW}(1;\vec{k}\,),\,(\tilde{\mu}-1)\chi_V+\delta\chi_V\Big]\,, \tag{B.12}$$

where χ_V is given by

$$\langle\vec{p}_1^{\,\prime}\vec{p}_2^{\,\prime}\,|\,\chi_V\,|\vec{p}_1\vec{p}_2\,\rangle=\tilde{\chi}_V(\vec{q},\vec{K},\vec{Q})(\vec{\tau}_1\cdot\vec{\tau}_2)\,\delta(\vec{q}_1+\vec{q}_2\,)\,, \tag{B.13}$$

$$i\tilde{\chi}_V(\vec{q},\vec{K},\vec{Q})=\frac{g^2_{PS}}{(2\pi)^3 64m^3}\Sigma^{(-)}(\vec{q},\,\vec{K})\ \tilde{\Delta}_{PS}(\vec{q}^{\,2})\,, \tag{B.14}$$

and $\delta\chi_V$ is the model-independent interaction boost which vanishes for a two-particle system with equal masses [4,16].

Evaluation of (B.12) gives for the boost contributions

$$\rho^{(2)}_{\chi_V}(2;\vec{k},\vec{q},\vec{K},\vec{Q}\,)_{PS}=(\tilde{\mu}-1)\frac{g^2_{PS}}{(2\pi)^3\,64m^3}\Big\{\,F^+_{e,1}\,(\vec{k}\times\vec{q}_2)\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)$$

$$-F^-_{e,1}\,(\vec{K}\times\vec{q}_2)\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)\Big\}\ \tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{B.15}$$

$$\vec{\jmath}^{\,(3)}_{\chi_V}(2;\vec{k},\vec{q},\vec{K},\vec{Q}\,)_{PS}=(\tilde{\mu}-1)\frac{g^2_{PS}}{(2\pi)^3\,128m^4}$$

$$\Big\{\,F^+_{e,1}\,\Big[\vec{Q}_1\,(\vec{k}\times\vec{q}_2)\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)+\vec{q}_2\,(\vec{K}\times\vec{q}_2)\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)\Big]$$

$$+G^+_{M,1}\,\vec{k}\times\Big[\vec{q}_2\times\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{K})-\vec{K}\times\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{q}_2)+i\vec{q}_2(\vec{\sigma}_2\cdot\vec{k})\Big]$$

$$
\begin{aligned}
&-F^-_{e,1}\left[\vec{Q}_1\,(\vec{K}\times\vec{q}_2)\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)+\vec{q}_2\,(\vec{k}\times\vec{q}_2)\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)\right]\\
&+G^-_{M,1}\,\vec{k}\times\Big[\vec{k}\times\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{q}_2)-\vec{q}_2\times\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{k})\\
&+i\vec{K}(\vec{\sigma}_2\cdot\vec{q}_2)-i\vec{q}_2(\vec{\sigma}_2\cdot\vec{K})\Big]\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,.
\end{aligned}
\tag{B.16}
$$

In the Breit frame, these expressions yield then

$$
\begin{aligned}
\rho^{(2)}_{\chi V}(2;\text{pro};\vec{k},\vec{q},\vec{Q})^+_{PS} &= (\tilde{\mu}-1)F^+_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,64m^3}\,(\vec{k}\times\vec{q}_2)\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})\\
&\quad+(1\leftrightarrow 2)\,,
\end{aligned}
\tag{B.17}
$$

$$
\begin{aligned}
\vec{\jmath}^{\,(3)}_{\chi V}(2;\text{pro};\vec{k},\vec{q},\vec{Q})^+_{PS} &= (\tilde{\mu}-1)\frac{g^2_{PS}}{(2\pi)^3\,128m^4}\Big\{F^+_{e,1}\,\vec{Q}\,(\vec{k}\times\vec{q}_2)\cdot(\vec{\sigma}_1\times\vec{\sigma}_2)\\
&\quad+iG^+_{M,1}\,\vec{k}\times\vec{q}_2\,(\vec{\sigma}_2\cdot\vec{k})\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,,
\end{aligned}
\tag{B.18}
$$

$$
\begin{aligned}
\vec{\jmath}^{\,(3)}_{\chi V}(2;\text{pro};\vec{k},\vec{q},\vec{Q})^-_{PS} &= -(\tilde{\mu}-1)\frac{g^2_{PS}}{(2\pi)^3\,128m^4}\Big\{F^-_{e,1}\,\vec{q}_2\,(\vec{\sigma}_1\times\vec{\sigma}_2)\cdot(\vec{k}\times\vec{q}_2)\\
&\quad+G^-_{M,1}\,\vec{k}\times[\vec{q}_2\times\vec{\sigma}_1\,(\vec{\sigma}_2\cdot\vec{k}\,)-\vec{k}\times\vec{\sigma}_1\,(\vec{\sigma}_2\cdot\vec{q}_2)]\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})\\
&\quad+(1\leftrightarrow 2)\,.
\end{aligned}
\tag{B.19}
$$

Alternative representation of χ_r and separation currents

We would like to remind the reader that the separation currents were introduced because for the one-body currents a useful cancellation occurs between them and the χ_r-ones. Also for the exchange currents it is convenient to consider the following combination of the χ_r, *sep* and *recoil* currents,

$$
\vec{\jmath}^{\,(3)}_{\chi sr}(2;\vec{k}\,)=\vec{\jmath}^{\,(3)}_{\chi_r}(2;\vec{k}\,)+\vec{\jmath}^{\,(3)}_{\text{sep}}(2;\vec{k}\,)-\vec{\jmath}^{\,(3)}_{\text{rec}}(2;\vec{k}\,)\,,
\tag{B.20}
$$

where the recoil current is given by

$$
\vec{\jmath}^{\,(3)}_{rec}(\vec{k})=-\frac{\vec{k}}{32m^2}\;\vec{k}\cdot\vec{\jmath}^{\,(1)}(\vec{k})\,.
\tag{B.21}
$$

Explicitly, one finds from (29), (43) and (B.21) the following expression for this combination

$$
\begin{aligned}
\vec{\jmath}^{\,(3)}_{\chi sr}(2;\vec{k}\,) &= \frac{\vec{k}}{32m^2}\,\vec{k}\cdot\vec{\jmath}^{\,(1)}(2;\vec{k}\,)+\frac{\vec{k}^{\,2}}{16m^2}\,\vec{\jmath}^{\,(1)}(2;\vec{k}\,)\\
&\quad+\frac{1}{32m^2}\Big[(\vec{k}\cdot\vec{q}_1)(\vec{k}\cdot\vec{\nabla}^{q_1}\,)+(\vec{k}\cdot\vec{q}_2)(\vec{k}\cdot\vec{\nabla}^{q_2}\,)\Big]\,\vec{\jmath}^{\,(1)}(2;\vec{k}\,)\,.
\end{aligned}
\tag{B.22}
$$

For a "mes"-type current, having the generic form

$$\vec{\jmath}^{\,(1)}(2;\mathrm{mes};\vec{k}\,) = (\vec{q}_1 - \vec{q}_2)\, f(\vec{q}_1^{\,2}, \vec{q}_2^{\,2})\, \mathrm{g}(\vec{q}_1, \vec{q}_2)\,, \tag{B.23}$$

the expression (B.22) becomes

$$\begin{aligned}
\vec{\jmath}^{\,(3)}_{\chi sr}(2;\vec{k}\,) = {} & \frac{\vec{k}}{16m^2}\, \vec{k}\cdot\vec{\jmath}^{\,(1)}(2;\mathrm{mes};\vec{k}\,) \\
& + \frac{1}{16m^2}\, g(\vec{q}_1, \vec{q}_2)\, \frac{\vec{q}}{(\vec{k}\cdot\vec{q}\,)}\, (\vec{k}\cdot\vec{q}_2)^2\, \tilde{\Delta}'_B(\vec{q}_2^{\,2}) \\
& + 2\,\vec{q}\, f(\vec{q}_1^{\,2}, \vec{q}_2^{\,2}) \Big[(\vec{k}\cdot\vec{q}_1)(\vec{k}\cdot\vec{\nabla}^{q_1}\,) + (\vec{k}\cdot\vec{q}_2)(\vec{k}\cdot\vec{\nabla}^{q_2}\,)\Big]\, g(\vec{q}_1, \vec{q}_2)\,.
\end{aligned} \tag{B.24}$$

For scalar and vector exchanges there is only a "mes"-type nonrelativistic exchange current given in (A.28) where $g(\vec{q}_1, \vec{q}_2)$ of (B.23) is just a constant. One finds without separating into "pro", "mes" and "ret" parts

$$\begin{aligned}
\vec{\jmath}^{\,(3)}_{\chi sr}(2;\vec{k},\,\vec{q},\,\vec{Q})^-_{\mathrm{B}} = {} & \frac{\vec{k}}{16m^2}\, \vec{k}\cdot\vec{\jmath}^{\,(1)}_F(2;\mathrm{mes};\vec{k},\vec{q}\,)^-_B \\
& \pm F^-_{e,1}\, \frac{g_B^2}{(2\pi)^3\, 16m^2}\, \vec{q}\, \frac{(\vec{k}\cdot\vec{q}_2)^2}{(\vec{k}\cdot\vec{q}\,)} \tilde{\Delta}'_B(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,,
\end{aligned} \tag{B.25}$$

where $B = S$ or V and the minus sign applies for vector exchange.

For pseudoscalar exchange one has to apply (B.22) and (B.24) to the nonrelativistic currents (A.57) and (A.58), respectively. Collecting all terms one finds

$$\begin{aligned}
\vec{\jmath}^{\,(3)}_{\chi sr}(2;\vec{k},\,\vec{q},\,\vec{Q})^-_{\mathrm{PS}} = {} & \vec{\jmath}^{\,(3)}_{sep}(2;\mathrm{mes};\vec{k},\vec{q},\vec{Q})^-_{PS} \\
& + F^-_{e,1}\, \frac{g_{PS}^2}{(2\pi)^3\, 64m^4} \Big[\vec{\sigma}_1 - (\vec{\sigma}_1\cdot\vec{q}_1)\frac{\vec{q}}{(\vec{k}\cdot\vec{q}\,)}\Big] (\vec{\sigma}_2\cdot\vec{q}_2)(\vec{k}\cdot\vec{q}_2)^2\, \tilde{\Delta}'_{PS}(\vec{q}_2^{\,2}) \\
& + F^-_{e,1}\, \frac{g_{PS}^2}{(2\pi)^3\, 128m^4}\, \vec{D}\, \tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,,
\end{aligned} \tag{B.26}$$

where $\vec{\jmath}^{\,(3)}_{sep}(2;\mathrm{mes};\vec{k},\vec{q},\vec{Q})^-_{PS}$ is given in (B.10) and

$$\begin{aligned}
\vec{D} = {} & \vec{\sigma}_1\Big[-2\vec{k}^{\,2}(\vec{\sigma}_2\cdot\vec{q}\,) + \Big(\frac{3}{2}\vec{k}^{\,2} - (\vec{k}\cdot\vec{q}\,)\Big)(\vec{\sigma}_2\cdot\vec{k}\,)\Big] - 2\vec{k}\,(\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}_2) \\
& - \vec{q}\Big[(\vec{\sigma}_1\cdot\vec{k}\,)(\vec{\sigma}_2\cdot\vec{q}_2) - (\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{k}\,)\Big]
\end{aligned} \tag{B.27}$$

$$\begin{aligned}
= {} & 2\vec{k}\,(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2) + \vec{q}_2\,\Sigma^{(+)}(\vec{k},\,\vec{q}_2) + \frac{1}{2}\vec{k}\Big[(\vec{\sigma}_1\cdot\vec{k}\,)(\vec{\sigma}_2\cdot\vec{q}_2) + (\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{k}\,)\Big] \\
& - \vec{k}\times\Big[\,2\,\vec{k}\times\vec{\sigma}_1\,(\vec{\sigma}_2\cdot\vec{q}_2) + \vec{q}_2\times\vec{\sigma}_1\,(\vec{\sigma}_2\cdot\vec{k}\,)\Big]\,,
\end{aligned} \tag{B.28}$$

The first form of $\vec{D}$ in (B.27) is more convenient for comparison with the results given earlier, whereas the second one in (B.28) is used in the appendix E in order to obtain the "pro-IV" current.

APPENDIX C: $\tilde{\mu}$-DEPENDENT CURRENTS

Collecting the $\tilde{\mu}$-dependent terms from (A.47), (A.48), and (B.17)-(B.19), we get for the total $\tilde{\mu}$-dependent current

$$
\begin{aligned}
\rho_{\tilde{\mu}}(2;\vec{k},\vec{q},\vec{Q}\,)_{PS} = {} & \tilde{\mu}\frac{g_{PS}^2}{(2\pi)^3\,32m^3}\Big[-\frac{1}{2}F_{e,1}^{+}\,\Sigma^{(+)}(\vec{q}_2,\,\vec{k}) + F_{e,1}^{-}\,\Sigma^{(+)}(\vec{q}_2,\,\vec{Q})\Big]\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) \\
& +(1\leftrightarrow 2)\,, \qquad\qquad (C.1) \\
\vec{\jmath}_{\tilde{\mu}}(2;\vec{k},\vec{q},\vec{K},\vec{Q}\,)_{PS} = {} & \frac{\vec{Q}_1}{2m}\rho_{\tilde{\mu}}(2;\vec{k},\vec{q},\vec{Q}\,)_{PS} \\
& +\tilde{\mu}\frac{g_{PS}^2}{(2\pi)^3\,64m^4}\Big\{\vec{q}_2\Big(\frac{1}{2}F_{e,1}^{-}\,\Sigma^{(+)}(\vec{q}_2,\,\vec{k}) - F_{e,1}^{+}\Sigma^{(+)}(\vec{q}_2,\,\vec{Q})\Big) \\
& +G_{M,1}^{+}\,\vec{k}\times\Big[\vec{q}_2\times\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{Q}) + \vec{Q}\times\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{q}_2) + \frac{i}{2}\vec{q}_2(\vec{\sigma}_2\cdot\vec{k})\Big] \\
& -G_{M,1}^{-}\,\vec{k}\times\Big[\frac{1}{2}\vec{q}_2\times\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{k}) + \frac{1}{2}\vec{k}\times\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{q}_2) + i\vec{q}_2(\vec{\sigma}_2\cdot\vec{Q}) \\
& +i\vec{Q}(\vec{\sigma}_2\cdot\vec{q}_2)\Big]\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,. \qquad\qquad (C.2)
\end{aligned}
$$

The charge density $\rho_{\tilde{\mu}}(2;\vec{k}\,)_{PS}$ is frame-independent and adds to the intrinsic charge density operator. The current $\vec{\jmath}_{\tilde{\mu}}(2;\vec{k},\vec{q},\vec{K},\vec{Q})_{PS}$ depends on the reference frame, i.e. on $\vec{K}$, only through $\vec{Q}_1 = \vec{Q} + \vec{K}/2$ in the first term of (C.2). Obviously,

$$
\vec{\jmath}_{\tilde{\mu}}(2;\vec{k},\vec{q},\vec{K},\vec{Q})_{PS} - \vec{\jmath}_{\tilde{\mu}}(2;\vec{k},\vec{q},\vec{0},\vec{Q})_{PS} = \frac{\vec{K}}{4m}\,\rho_{\tilde{\mu}}(2;\vec{k},\vec{q},\vec{Q}\,)_{PS}\,, \qquad (C.3)
$$

which is consistent with the general expression (4). Furthermore, for the divergence of the current one easily finds from (C.1) and (C.2)

$$
\begin{aligned}
\vec{k}\cdot\vec{\jmath}_{\tilde{\mu}}(2;\vec{k},\vec{0})_{PS} = {} & \frac{(\vec{q}\cdot\vec{Q})}{m}\rho_{\tilde{\mu}}(2;\vec{k}\,)_{PS} \\
& +\tilde{\mu}\frac{g_{PS}^2}{(2\pi)^3\,32m^4}\Big\{-\frac{1}{2}F_{e,1}^{+}\Big[(\vec{k}\cdot\vec{q}_2)\Sigma^{(+)}(\vec{q}_2,\,\vec{Q}) + (\vec{Q}\cdot\vec{q}_2)\Sigma^{(+)}(\vec{q}_2,\,\vec{k})\Big] \\
& +F_{e,1}^{-}\Big[(\vec{Q}\cdot\vec{q}_2)\Sigma^{(+)}(\vec{q}_2,\,\vec{Q}) + \frac{1}{4}(\vec{k}\cdot\vec{q}_2)\Sigma^{(+)}(\vec{q}_2,\,\vec{k})\Big]\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) + (1\leftrightarrow 2) \\
= {} & \langle\vec{p}\,'\,|\Big[t^{(1)}\,,\,\rho_{\tilde{\mu}}(2;\vec{k}\,)_{PS}\Big] + \Big[v_{\tilde{\mu}}^{(3)}\,,\,\rho^{(0)}(1;\vec{k}\,)\Big]\,|\vec{p}\,\rangle\,, \qquad\qquad (C.4)
\end{aligned}
$$

where the $\tilde{\mu}$-dependent intrinsic potential $\tilde{v}^{(3)}_{\tilde{\mu}}(\vec{q},\vec{Q})$ is given in (A.44).

According to (A.46) and (B.12), the $\tilde{\mu}$-dependent operators of (A.44), (C.1) and (C.2) as well as the foregoing continuity equation can be obtained from the nonrelativistic one-nucleon current and the kinetic energy with the help of another (approximate) unitary transformation $(1 - i\tilde{\mu}U_{\tilde{\mu}})$ [18], where

$$U_{\tilde{\mu}} = U_{PS} - \chi_V \,. \tag{C.5}$$

In detail, this gives for $U_{\tilde{\mu}}$

$$\langle \vec{p}_1{}'\vec{p}_2{}' \,|\, U_{\tilde{\mu}} \,|\vec{p}_1\vec{p}_2 \,\rangle = \tilde{U}_{\tilde{\mu}}(\vec{q},\vec{Q})(\vec{\tau}_1\cdot\vec{\tau}_2)\,\delta(\vec{q}_1+\vec{q}_2\,)\,, \tag{C.6}$$

$$i\tilde{U}_{\tilde{\mu}}(\vec{q},\vec{Q}) = -\frac{g^2_{PS}}{(2\pi)^3\,32m^3}\,\Sigma^{(+)}(\vec{q},\,\vec{Q})\;\tilde{\Delta}_{PS}(\vec{q}^{\,2})\,, \tag{C.7}$$

where we already have used the fact that there is no $\vec{K}$-dependence. Also for more than two nucleons the difference $U_{\tilde{\mu}} = U_{PS} - \chi_V$ gives the operator depending only on the relative coordinates $\vec{p}_a - \frac{m}{M}\vec{P}$.

One can use any unitary transformation acting in the intrinsic space only with the generator $U = \mathcal{O}(1/m^2)$ in two ways. First, one can apply it only to the intrinsic Hamiltonian and currents. The transformed operators still obey the intrinsic continuity equation. Then, inserting these into the full operators (1)-(4) and inspecting the $\vec{K}$-dependent terms, one finds that only the convection current $(\vec{K}/2M)\,\rho^{(2)}(\vec{k}\,)$ is affected by the unitary transformation resulting in an additional contribution $(\vec{K}/2M)\,\rho^{(2)}_U(2;\vec{k}\,)$ with

$$\rho^{(2)}_U(2;\vec{k}\,) = i\Big[\,\rho^{(0)}(1;\vec{k}\,)\,,\,U\,\Big]\,. \tag{C.8}$$

Since the general parametrization (1)-(4) is preserved, the full current is still conserved and transformes properly under the Poincaré transformation. Alternatively, one may let act the transformation in the full Hilbert space including the c.m. motion part. That is, one transforms also the nonrelativistic convection current $(\vec{K}/2M)\,\rho^{(0)}(\vec{k}\,)$ and gets the correction to its relativistic part as given by (C.8). Both views are equivalent and consistent, and it implies that it is sufficient to transform the conserved intrinsic current only. Another example of such a unitary transformation is encountered in the next appendix.

APPENDIX D: RETARDATION CURRENTS FOR $\nu \neq 1/2$

In the framework of the $1/m$-expansion techniques [2], the effects of retardation of the exchanged mesons are included via the Taylor expansion of the meson propagators (see also (A.20) and (A.24))

$$\tilde{\Delta}_B(\vec{q}^{\,2} - q_0^2) \simeq \tilde{\Delta}_B(\vec{q}^{\,2}) - q_0^2\, \tilde{\Delta}'_B(\vec{q}^{\,2})\,, \tag{D.1}$$

and expressing the meson energy in terms of the energies of the on-mass-shell nucleons. This procedure is unambiguous for the nonrelativistic reduction of the Feynman amplitudes corresponding to the MEC operators, but in order to define the general nuclear potential one has to allow for an off-energy-shell continuation of the corresponding amplitude, and then q_0 is no longer fixed by energy conservation at the vertex. It has been argued in [3] that up to the order considered the most general expression symmetric with respect to nucleon interchange reads

$$q_0^2 \to -q_{10}\, q_{20} + \frac{1-\nu}{2}(q_{10} + q_{20})^2 = \frac{1}{4m^2}\Big[(\vec{q}\cdot\vec{P})^2 - 2\tilde{\nu}(\vec{q}\cdot\vec{Q})^2\Big] + \mathcal{O}(m^{-4})\,, \tag{D.2}$$

where ν is an arbitrary parameter and $\tilde{\nu} = \nu - 1/2$. In [3] it is described in detail how this freedom translates into the ν-dependent retardation contribution to the MECs from the positive-energy nuclear pole Born diagram.

It is convenient to introduce the following notation

$$\tilde{w}_S^{(1)}(\vec{q}\,) = -\frac{g_S^2}{(2\pi)^3}\, \tilde{\Delta}'_S(\vec{q}^{\,2}), \tag{D.3}$$

$$\tilde{w}_V^{(1)}(\vec{q}\,) = \frac{g_V^2}{(2\pi)^3}\, \tilde{\Delta}'_V(\vec{q}^{\,2}), \tag{D.4}$$

$$\tilde{w}_{PS}^{(1)}(\vec{q}\,) = -\frac{g_{PS}^2}{(2\pi)^3\, 4m^2}(\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}\,)\, \tilde{\Delta}'_{PS}(\vec{q}^{\,2}). \tag{D.5}$$

The retardation contribution to the potential in an arbitrary frame is then given by

$$\tilde{v}^{(3)}_{B,ret}(\vec{q},\vec{Q},\vec{P}) = -\frac{1}{4m^2}\, \tilde{w}_B^{(1)}(\vec{q}\,)\, \Big[(\vec{q}\cdot\vec{P})^2 - 2\tilde{\nu}(\vec{q}\cdot\vec{Q})^2\Big]\,, \tag{D.6}$$

where the first $\vec{P}$-dependent term is required by the Foldy condition and the second one is the retardation contribution to the intrinsic potential

$$\tilde{v}^{(3)}_{B,\tilde{\nu}}(\vec{q},\vec{Q}) = \frac{\tilde{\nu}}{2m^2}\; \tilde{w}^{(1)}_B(\vec{q})\; (\vec{q}\cdot\vec{Q})^2 \,. \tag{D.7}$$

Obviously, (D.7) vanishes for $\tilde{\nu} = 0$, i.e. $\nu = 1/2$, which is the choice for the intrinsic potentials and the retardation e.m. operators adopted in the main part of the paper. It is also common for the construction of realistic NN-potentials which usually do not include the retardation terms in the c.m. frame. In this appendix we present for completeness the currents for arbitrary $\tilde{\nu}$.

Since the expressions for the retardation operators ATA-(4.21-4.24) contain a number of misprints and do not contain the strong form factors, we find it useful to present the correct form here

$$\begin{aligned}
\rho^{(2)}_{FW}(2;\mathrm{ret};\vec{k}\,)_B &= -\frac{1}{4m}\; \tilde{w}^{(1)}_B(\vec{q}_2\,)\left(F^+_{e,1}R_k - F^-_{e,1}R_Q\right) + (1\leftrightarrow 2)\,, && \text{(D.8)}\\
\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{ret};\vec{k}\,)_B &= -\frac{1}{8m^2}\; \tilde{w}^{(1)}_B(\vec{q}_2\,)\Big(F^+_{e,1}[R_k\vec{Q}_1 + R_Q\vec{q}_2]\\
&\qquad -F^-_{e,1}[R_Q\vec{Q}_1 + R_k\vec{q}_2]\Big) + (1\leftrightarrow 2) + \vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{ret\text{-}tr};\vec{k}\,)_B\\
&= \vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{ret\text{-}tr};\vec{k}\,)_B + \frac{\vec{Q}_1}{2m}\rho^{(2)}_{FW}(2;\mathrm{ret};\vec{k}\,)_B\\
&\qquad -\frac{1}{8m^2}\; \tilde{w}^{(1)}_B(\vec{q}_2\,)\,\vec{q}_2\left(F^+_{e,1}R_Q - F^-_{e,1}R_k\right) + (1\leftrightarrow 2)\,, && \text{(D.9)}\\
\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{ret\text{-}tr};\vec{k}\,)_{S,V} &= -\frac{1}{8m^2}\; \tilde{w}^{(1)}_{S,V}(\vec{q}_2\,)\left(G^+_{M,1}R_k - G^-_{M,1}R_Q\right)i(\vec{\sigma}_1\times\vec{k}) + (1\leftrightarrow 2)\,, && \text{(D.10)}\\
\vec{\jmath}^{\,(3)}_{FW}(2;\mathrm{ret\text{-}tr};\vec{k}\,)_{PS} &= -\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\,i\vec{k}\times\Big\{G^+_{M,1}[R_k\vec{q}_2 + R_Q i\vec{\sigma}_1\times\vec{q}_2]\\
&\qquad -G^-_{M,1}[R_Q\vec{q}_2 + R_k i\vec{\sigma}_1\times\vec{q}_2]\Big\}\,(\vec{\sigma}_2\cdot\vec{q}_2)\tilde{\Delta}'_{PS}(\vec{q}^{\,2}_2) + (1\leftrightarrow 2)\,, && \text{(D.11)}
\end{aligned}$$

with

$$\begin{aligned}
R_k &= (\frac{1}{2} - \tilde{\nu})(\vec{k}\cdot\vec{q}_2)\,, && \text{(D.12)}\\
R_Q &= \frac{1}{2}(\vec{Q}_1 + 3\vec{Q}_2)\cdot\vec{q}_2 - \tilde{\nu}\,(\vec{Q}_1 - \vec{Q}_2)\cdot\vec{q}_2\,. && \text{(D.13)}
\end{aligned}$$

The $\tilde{\nu}$-independent part of these operators is included in the retardation currents in Appendix A and then added to the intrinsic currents in the main part of the paper. Separating the $\tilde{\nu}$-dependent part we get

$$\rho^{(2)}_{\tilde{\nu}}(2;\mathrm{ret};\vec{k}\,)_B = \frac{\tilde{\nu}}{4m}\; \tilde{w}^{(1)}_B(\vec{q}_2\,)\left[F^+_{e,1}(\vec{k}\cdot\vec{q}_2) - 2F^-_{e,1}(\vec{Q}\cdot\vec{q}_2)\right] + (1\leftrightarrow 2)\,, \tag{D.14}$$

$$
\begin{aligned}
\vec{\jmath}^{\,(3)}_{\tilde{\nu}}(2;\text{ret};\vec{k},\vec{K})_B &= \vec{\jmath}_{\tilde{\nu}}(2;\text{ret-tr};\vec{k}\,)_B + \frac{\vec{Q}_1}{2m}\rho^{(2)}_{\tilde{\nu}}(2;\text{ret};\vec{k}\,)_B \\
&\quad + \frac{\tilde{\nu}}{8m^2}\;\tilde{w}^{(1)}_B(\vec{q}_2\,)\,\vec{q}_2\left(2F^+_{e,1}\,(\vec{Q}\cdot\vec{q}_2) - F^-_{e,1}\,(\vec{k}\cdot\vec{q}_2)\right) + (1\leftrightarrow 2)\,, \qquad \text{(D.15)} \\
\vec{\jmath}^{\,(3)}_{\tilde{\nu}}(2;\text{ret-tr};\vec{k}\,)_{S,V} &= \frac{\tilde{\nu}}{8m^2}\;\tilde{w}^{(1)}_{S,V}(\vec{q}_2\,)\left[G^+_{M,1}\,(\vec{k}\cdot\vec{q}_2) - 2G^-_{M,1}(\vec{Q}\cdot\vec{q}_2)\right] i\vec{\sigma}_1\times\vec{k} \\
&\quad + (1\leftrightarrow 2)\,, \qquad \text{(D.16)} \\
\vec{\jmath}^{\,(3)}_{\tilde{\nu}}(2;\text{ret-tr};\vec{k}\,)_{PS} &= \tilde{\nu}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\,i\vec{k}\times\Big\{G^+_{M,1}\Big[\vec{q}_2\,(\vec{k}\cdot\vec{q}_2) + 2i\vec{\sigma}_1\times\vec{q}_2(\vec{Q}\cdot\vec{q}_2)\Big] \\
&\quad - G^-_{M,1}\Big[2\vec{q}_2\,(\vec{Q}\cdot\vec{q}_2) + i\vec{\sigma}_1\times\vec{q}_2\,(\vec{k}\cdot\vec{q}_2)\Big]\Big\}\,(\vec{\sigma}_2\cdot\vec{q}_2)\tilde{\Delta}'_{PS}(\vec{q}^{\,2}_2) \\
&\quad + (1\leftrightarrow 2)\,. \qquad \text{(D.17)}
\end{aligned}
$$

It is easy to show that

$$
\vec{k}\cdot\vec{\jmath}^{\,(3)}_{\tilde{\nu}}(2;\text{ret};\vec{k},\vec{0})_B = \langle\vec{p}\,'\,|\Big[t^{(1)},\,\rho^{(2)}_{\tilde{\nu}}(2;\text{ret};\vec{k}\,)_B\Big] + \Big[v^{(3)}_{B,\tilde{\nu}}\,,\,\rho^{(0)}(1;\vec{k}\,)\Big]|\vec{p}\,\rangle\,, \qquad \text{(D.18)}
$$

because one finds for the commutator

$$
\begin{aligned}
\langle\vec{p}\,'\,|\Big[\tilde{v}^{(3)}_{B,\tilde{\nu}}\,,\,\rho^{(0)}(1;\vec{k}\,)\Big]|\vec{p}\,\rangle &= \frac{\tilde{\nu}}{2m^2}\;\tilde{w}^{(1)}_B(\vec{q}_2\,)\Big(F^+_{e,1}(\vec{k}\cdot\vec{q}_2)(\vec{Q}\cdot\vec{q}_2) - F^-_{e,1}((\vec{Q}\cdot\vec{q}_2)^2 + \frac{1}{4}(\vec{k}\cdot\vec{q}_2)^2)\Big) \\
&\quad + (1\leftrightarrow 2) \qquad \text{(D.19)}
\end{aligned}
$$

One can also check that, similar to the $\tilde{\mu}$-dependent ones discussed at the end of the previous appendix, all $\tilde{\nu}$-dependent operators can be generated with the help of a unitary transformation $(1 - iU_{B,\tilde{\nu}})$ from the nonrelativistic one-nucleon currents and the kinetic energy

$$
\begin{aligned}
j^{(3)}_{\lambda,\tilde{\nu}}(2;\vec{k}\,)_B &= i\Big[j^{(1)}_{\lambda,FW}(1;\vec{k}\,)\,,\,U_{B,\tilde{\nu}}\Big]\,, \qquad \text{(D.20)} \\
\tilde{v}^{(3)}_{B,\tilde{\nu}} &= i\Big[\frac{\vec{p}^{\,2}}{m}\,,\,U_{B,\tilde{\nu}}\Big]\,, \qquad \text{(D.21)}
\end{aligned}
$$

where

$$
\begin{aligned}
i\tilde{U}_{B,\tilde{\nu}}(\vec{q},\vec{Q}) &= \tilde{\nu}\frac{(\vec{q}\cdot\vec{Q})}{2m}\;\;\tilde{w}^{(1)}_B(\vec{q}\,) \\
&= \frac{\tilde{\nu}}{2}\langle\vec{p}\,'\,|\Big[t^{(1)},\;\;\tilde{w}^{(1)}_B(\vec{q}\,)\Big]|\vec{p}\,\rangle\,. \qquad \text{(D.22)}
\end{aligned}
$$

APPENDIX E: CONTINUITY EQUATION FOR THE F^- CURRENTS FOR PSEUDOSCALAR EXCHANGE

In this appendix we want to identify the various pieces of the "pro" and "mes" currents corresponding to the various commutators in (68). To this end we split the total "pro" and "mes" currents into several parts (labelled by I, II,..) which will be specified in the following

$$\vec{\jmath}_F^{(3)}(2;\text{pro};\vec{k},\vec{q},\vec{Q})^-_{PS} = \sum_{i=I}^{V} \vec{\jmath}_F^{(3)}(2;\text{pro-i};\vec{k},\vec{q},\vec{Q})^-_{PS}\,, \tag{E.1}$$

$$\vec{\jmath}_F^{(3)}(2;\text{mes};\vec{k},\vec{q},\vec{Q})^-_{PS} = \sum_{i=I}^{IV} \vec{\jmath}_F^{(3)}(2;\text{mes-i};\vec{k},\vec{q},\vec{Q})^-_{PS}\,. \tag{E.2}$$

As first current, we single out the purely transverse G_M^- term since it drops out from the continuity equation

$$\begin{aligned}\vec{\jmath}_F^{(3)}(2;\text{pro-V};\vec{k},\vec{q},\vec{Q})^-_{PS} = G^-_{M,1}\,\frac{g_{PS}^2}{(2\pi)^3\,128m^4}\,\vec{k}\times\Big[\Big(6i\,\vec{Q}+5\,\vec{k}\times\vec{\sigma}_1\Big)(\vec{\sigma}_2\cdot\vec{q}_2)\\ +\vec{q}_2\times\vec{\sigma}_1\,(\vec{\sigma}_2\cdot\vec{k}\,)-2i\vec{q}_2\,(\vec{\sigma}_2\cdot\vec{Q})\Big]\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,.\end{aligned} \tag{E.3}$$

Now we start with the first commutator in (68) which has two contributions from the "pro" and "mes" parts of $\rho_F^{(2)}(2;\vec{k}\,)$, namely

$$\begin{aligned}\langle\vec{p}\,'\,|\Big[t^{(1)},\,\rho_F^{(2)}(2;\text{pro};\vec{k}\,)\Big]^-|\vec{p}\,\rangle = F^-_{e,1}\frac{g_{PS}^2}{(2\pi)^3\,32m^4}(\vec{q}\cdot\vec{Q})\,\Sigma^{(+)}(\vec{q}_2,\,\vec{Q})\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})\\ +(1\leftrightarrow 2)\end{aligned} \tag{E.4}$$

and

$$\langle\vec{p}\,'\,|\Big[t^{(1)},\,\rho_F^{(2)}(2;\text{mes};\vec{k}\,)\Big]^-|\vec{p}\,\rangle = -F^-_{e,1}\frac{g_{PS}^2}{(2\pi)^3\,8m^4}(\vec{k}\cdot\vec{Q})(\vec{q}\cdot\vec{Q})(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,. \tag{E.5}$$

Their current counterparts are easy to identify, namely

$$\begin{aligned}\vec{\jmath}_F^{(3)}(2;\text{pro-I};\vec{k},\vec{q},\vec{Q})^-_{PS} = -F^-_{e,1}\,\frac{g_{PS}^2}{(2\pi)^3\,64m^4}\Big\{\vec{Q}\,\Sigma^{(-)}(\vec{Q},\vec{q}_2)\,+2\,\vec{\sigma}_1\,(\vec{Q}\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{Q})\,\Big\}\\ \tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,,\end{aligned} \tag{E.6}$$

$$\begin{aligned}\vec{\jmath}_F^{(3)}(2;\text{mes-I};\vec{k},\vec{q},\vec{Q})^-_{PS} = F^-_{e,1}\,\frac{g_{PS}^2}{(2\pi)^3\,16m^4}\,\vec{q}\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\\ \Big\{(\vec{q}_2\cdot\vec{Q})(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{Q})+(\vec{q}_1\cdot\vec{Q})(\vec{\sigma}_1\cdot\vec{Q})(\vec{\sigma}_2\cdot\vec{q}_2)\Big\}\,,\end{aligned} \tag{E.7}$$

with the divergence

$$\vec{k}\cdot\Big(\vec{\jmath}_F^{\,(3)}(2;\text{pro-I};\vec{k},\vec{q},\vec{Q})^-_{PS}+\vec{\jmath}_F^{\,(3)}(2;\text{mes-I};\vec{k},\vec{q},\vec{Q})^-_{PS}\Big)=\langle\vec{p}^{\,\prime}\,|\Big[t^{(1)},\,\rho_F^{(2)}(2;\text{pro};\vec{k}\,)\Big]^-|\vec{p}\,\rangle\,, \tag{E.8}$$

and

$$\vec{\jmath}_F^{\,(3)}(2;\text{mes-II};\vec{k},\vec{q},\vec{Q})^-_{PS}=-F^-_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,8m^4}\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\,\frac{\vec{q}}{(\vec{k}\cdot\vec{q}\,)}\,(\vec{k}\cdot\vec{Q})(\vec{q}\cdot\vec{Q})(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)\,, \tag{E.9}$$

with

$$\vec{k}\cdot\vec{\jmath}_F^{\,(3)}(2;\text{mes-II};\vec{k},\vec{q},\vec{Q})^-_{PS}=\langle\vec{p}^{\,\prime}\,|\Big[t^{(1)},\,\rho_F^{(2)}(2;\text{mes};\vec{k}\,)\Big]^-|\vec{p}\,\rangle\,. \tag{E.10}$$

Furthermore, the divergence of the "recoil" current

$$\vec{\jmath}_F^{\,(3)}(2;\text{pro-II};\vec{k},\vec{q},\vec{Q})^-_{PS}=-F^-_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,128m^4}\,\vec{k}\,(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2)\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,, \tag{E.11}$$

yields the recoil contribution to (68), i.e.

$$\vec{k}\cdot\vec{\jmath}_F^{\,(3)}(2;\text{pro-II};\vec{k},\vec{q},\vec{Q})^-_{PS}=-\frac{\vec{k}^{\,2}}{32m^2}\,\langle\vec{p}^{\,\prime}\,|\Big[v^{(1)}_{PS},\,\rho_F^{(0)}(1;\vec{k}\,)\Big]^-|\vec{p}\,\rangle\,. \tag{E.12}$$

Of the two remaining commutators of (68) we will first consider the commutator of the nonrelativistic potential with the relativistic charge density $\rho^{(2)}_{F,e}(1;\vec{k}\,)$

$$\begin{aligned}\langle\vec{p}^{\,\prime}\,|\Big[v^{(1)}_{PS}\,,\,\rho^{(2)}_{F,e}(1;\vec{k}\,)\Big]^-|\vec{p}\,\rangle=-F^-_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,64m^4}\Big\{&i\vec{k}\cdot(\vec{q}_2\times\vec{Q})(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}_2+\vec{q}_2^{\,2}\Sigma^{(+)}(\vec{k},\vec{q}_2)\\&+2(\vec{k}\cdot\vec{q}_1)(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2)\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2})+(1\leftrightarrow 2)\,.\end{aligned} \tag{E.13}$$

Recall that the current satisfying the continuity equation with the similar commutator of the κ-part $[v^{(1)}_{PS}\,,\,\rho^{(2)}_{F,\kappa}(1;\vec{k}\,)]$ has been absorbed into the single-nucleon current, as discussed in the Sect. IIIA. Explicitly, it is given in (ATA-4.17b). Note that this current does not change in the transition from the FW currents to the intrinsic ones, since $\rho^{(2)}_{FW,\kappa}(1;\vec{k}\,)=\rho^{(2)}_{F,\kappa}(1;\vec{k}\,)$. We can get the corresponding FW current proportional to $\hat{e}_1$ replacing $F^-_{\kappa,1}\to F^-_{e,1}/2$ in (ATA-4.17b) giving only a "pro" contribution

$$
\begin{aligned}
\vec{\jmath}^{\,(3)}_{FW,e}(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})^{-}_{PS} = -F^{-}_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\Big[\vec{q}_1\,(\vec{\sigma}_1\cdot\vec{q}_2) \\
+\vec{\sigma}_1\,\vec{q}_2^{\,2} + i\vec{q}_2\times\vec{Q}\Big]\,(\vec{\sigma}_2\cdot\vec{q}_2)\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,. \qquad \text{(E.14)}
\end{aligned}
$$

But since $\rho^{(2)}_{F,e}(1;\vec{k}\,) = \rho^{(2)}_{FW,e}(1;\vec{k}\,) + \rho^{(2)}_{\chi_\sigma}(1;\vec{k}\,)$ one has to add to (E.14) in this case the χ_σ currents from (B.5) and (B.6) in order to obtain the intrinsic "pro" and "mes" currents saturating (E.13)

$$
\begin{aligned}
\vec{\jmath}^{\,(3)}_{F}(2;\mathrm{pro\text{-}III};\vec{k},\vec{q},\vec{Q})^{-}_{PS} &= \vec{\jmath}^{\,(3)}_{FW,e}(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})^{-}_{PS} + \vec{\jmath}^{\,(3)}_{\chi_\sigma}(2;\mathrm{pro};\vec{k},\vec{q},\vec{Q})^{-}_{PS}\,, \\
&= -F^{-}_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,64m^4}\Big\{\vec{\sigma}_1\Big[(2\vec{q}_2^{\,2} - (\vec{k}\cdot\vec{q}_2))(\vec{\sigma}_2\cdot\vec{q}_2) + \vec{q}_2^{\,2}(\vec{\sigma}_2\cdot\vec{k}\,) \\
&\quad -i\vec{k}\times\vec{q}\cdot\vec{Q}\,\Big] + \vec{q}\,\Big[\,2(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2) + (\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{k}\,)\Big] \\
&\quad +\Big[\vec{k}\,\vec{\sigma}_1\cdot(2\vec{q}_2 - \frac{1}{2}\vec{k}\,) + 2i(\vec{k}-\vec{q}\,)\times\vec{Q}\,\Big](\vec{\sigma}_2\cdot\vec{q}_2)\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) \\
&\quad +(1\leftrightarrow 2) \qquad \text{(E.15)} \\
&= -F^{-}_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,64m^4}\Big\{\vec{\sigma}_1\Big[2(\vec{q}\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}\,) + (\vec{q}_2^{\,2} - (\vec{q}\cdot\vec{q}_2))(\vec{\sigma}_2\cdot\vec{k}\,) \\
&\quad -i\vec{k}\times\vec{q}\cdot\vec{Q}\,\Big] + 2\vec{q}_1\,(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2) + \vec{q}\,(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{k}\,) \\
&\quad -\vec{k}\,(\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}_2) + 2i(\vec{k}-\vec{q}\,)\times\vec{Q}\,\Big](\vec{\sigma}_2\cdot\vec{q}_2)\Big\}\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) \\
&\quad +(1\leftrightarrow 2)\,, \qquad \text{(E.16)} \\
\vec{\jmath}^{\,(3)}_{F}(2;\mathrm{mes\text{-}III};\vec{k},\vec{q},\vec{Q})^{-}_{PS} &= F^{-}_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\,\vec{q}\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2}) \\
&\quad \Big\{\vec{q}^{\,2}\,(\vec{\sigma}_1\cdot\vec{k}\,)(\vec{\sigma}_2\cdot\vec{k}\,) + \vec{k}^{\,2}\,(\vec{\sigma}_1\cdot\vec{q}\,)(\vec{\sigma}_2\cdot\vec{q}\,) \\
&\quad +\vec{k}\times\vec{q}\cdot\Big[(\vec{\sigma}_1\times\vec{\sigma}_2\,)(\vec{q}_1\cdot\vec{q}_2) - i\vec{Q}\,((\vec{\sigma}_1\cdot\vec{q}_1) + (\vec{\sigma}_2\cdot\vec{q}_2))\Big]\Big\}\,. \qquad \text{(E.17)}
\end{aligned}
$$

The first form (E.15) is convenient for reconstructing the total "pro" current, the second one (E.16) for verifying the continuity equation.

Finally, the currents belonging to the remaining commutator of (68)

$$
\begin{aligned}
\langle\vec{p}\,'\,|\Big[v^{(3)}_{PS},\,\rho^{(0)}_{F}(1;\vec{k}\,)\Big]^{-}|\vec{p}\,\rangle = -F^{-}_{e,1}\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\Big[2\vec{Q}^{\,2} + 2\vec{q}_2^{\,2} + \frac{1}{2}\vec{k}^{\,2}\Big] \\
(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2)\;\tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,, \qquad \text{(E.18)}
\end{aligned}
$$

are made up from the remaining pieces of the total "pro" and "mes" currents that have not yet been singled out. Let us write them in the form

$$
\begin{aligned}
\vec{\jmath}_F^{(3)}(2;\text{pro-IV};\vec{k},\vec{q},\vec{Q})^-_{PS} &= -F^-_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\Big\{\Big[\,2\,\vec{\sigma}_1\,(\vec{q}_2^{\,2}+\vec{Q}^{\,2}) + \frac{1}{2}\vec{k}\,(\vec{\sigma}_1\cdot\vec{q}_2) + 2\vec{q}\,(\vec{\sigma}_1\cdot\vec{q}_1) \\
&\quad + i\vec{k}\times\vec{Q}\Big](\vec{\sigma}_2\cdot\vec{q}_2) - \frac{1}{8}\vec{k}\,\Big[(\vec{\sigma}_1\cdot\vec{k}\,)(\vec{\sigma}_2\cdot\vec{q}_2) + (\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{k}\,)\Big] \\
&\quad + \frac{1}{2}\vec{k}\times\Big[\,\vec{k}\times\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{q}_2) + \frac{1}{2}\vec{q}_2\times\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{k}\,)\Big]\Big\} \\
&\quad \tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) + (1\leftrightarrow 2) \qquad\qquad \text{(E.19)} \\
&= -F^-_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\Big\{\,\vec{\sigma}_1\,\Big[\,(2\vec{q}_2^{\,2} + 2\vec{Q}^{\,2} - \frac{1}{2}\vec{k}^{\,2})(\vec{\sigma}_2\cdot\vec{q}_2) \\
&\quad - \frac{1}{4}(\vec{k}\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{k}\,)\Big] + i\vec{k}\times\vec{Q}(\vec{\sigma}_2\cdot\vec{q}_2) \\
&\quad + \vec{q}\Big[\,2(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2) - \frac{1}{4}(\vec{\sigma}_1\cdot\vec{k}\,)(\vec{\sigma}_2\cdot\vec{k}\,)\Big] \\
&\quad + \vec{k}\Big[\,\frac{1}{2}(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{q}_2) + \frac{3}{8}(\vec{\sigma}_1\cdot\vec{k}\,)(\vec{\sigma}_2\cdot\vec{q}_2) \\
&\quad + \frac{1}{8}(\vec{\sigma}_1\cdot\vec{q}_2)(\vec{\sigma}_2\cdot\vec{k}\,)\Big]\Big\}\,\tilde{\Delta}_{PS}(\vec{q}_2^{\,2}) + (1\leftrightarrow 2)\,, \qquad\qquad \text{(E.20)} \\
\vec{\jmath}_F^{(3)}(2;\text{mes-IV};\vec{k},\vec{q},\vec{Q})^-_{PS} &= F^-_{e,1}\,\frac{g^2_{PS}}{(2\pi)^3\,32m^4}\,\vec{q}\,f(\vec{q}_1^{\,2},\vec{q}_2^{\,2})\Big\{\Big[4\vec{q}^{\,2} + 4\vec{Q}^{\,2} + \vec{k}^{\,2}\Big](\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2) \\
&\quad - \frac{\vec{k}^{\,2}}{4}\Big[(\vec{\sigma}_1\cdot\vec{k}\,)(\vec{\sigma}_2\cdot\vec{q}_2) + (\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{k}\,)\Big]\Big\}\,. \qquad\qquad \text{(E.21)}
\end{aligned}
$$

Again, the first form (E.19) is more convenient for checking the continuity equation, the second one (E.20) for constructing the the total current of (129).

TABLES

TABLE I. Survey on the contributons to the two-body charge density $\rho(2;\vec{k}\,)$ with a listing of the corresponding equations.

meson	type	$\rho_{FW}^{(2)\,+}$	$\rho_{\chi V}^{(2)\,+}$	$\rho_{F}^{(2)\,+}$	$\rho_{F}^{(2)\,-}$
S	mes				(85)
S	ret	(A.26)		(79)	(86)
V	mes				(85)
V	ret	(A.26)		(79)	(86)
V	mes-tr				(105)
PS	pro	(A.53)	(B.17)	(117)	(126)
PS	mes				(127)
PS	ret	(A.55)		(119)	(128)

TABLE II. Survey on the contributions to the two-body current density $\vec{\jmath}(2;\vec{k}\,)$ with a listing of the corresponding equations.

meson	type	$\vec{\jmath}_F^{\,(1)\,-}$	$\vec{\jmath}_{FW}^{\,(3)\,+}$	$\vec{\jmath}_{\chi V}^{\,(3)\,+}$	$\vec{\jmath}_F^{\,(3)\,+}$	$\vec{\jmath}_{FW}^{\,(3)\,-}$	$\vec{\jmath}_{sep}^{\,(3)\,-}$	$\vec{\jmath}_{\chi r}^{\,(3)\,-}$	$\vec{\jmath}_{\chi \sigma}^{\,(3)\,-}$	$\vec{\jmath}_{\chi V}^{\,(3)\,-}$	$\vec{\jmath}_F^{\,(3)\,-}$
S	pro		(A.25)		(78)	(A.29)		(B.2)			(87)
S	mes	(A.28)				(A.31)			(B.1)		(88)
S	ret		(A.27)		(80)	(A.33)	(B.4)	(B.3)			(89)
V	pro		(A.34)		(100)	(A.35)		(B.2)			(103)
V	mes	(A.28)				(A.36)			(B.1)		(104)
V	ret		(A.27)		(80)	(A.33)	(B.4)	(B.3)			(89)
V	mes-tr					(A.38)					(106)
PS	pro	(A.57)	(A.54)	(B.18)	(118)	(A.61)	(B.9)	(B.7)	(B.5)	(B.19)	(129)
PS	mes	(A.58)				(A.63)	(B.10)		(B.6)		(130)
PS	ret		(A.56)		(120)	(A.65)	(B.11)	(B.8)			(131)

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